Cut-off Space of a Gyromagnetic Planar Disk Resonator with a Triplet of Stubs with Up and Down Magnetization

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**Abstract—** A switchable circulator may be realized by internally or externally switching or latching a suitable ferrite resonator at the junction of three transmission lines or waveguides. This paper investigates the split frequencies and therefore the quality factor of a geometry consisting of a half wave-long cylindrical cavity and another which consists of a tri-toroidal structure. The model adopted in this paper divides the surface of the resonator into up and down uniformly magnetized but unequal regions. The boundary, between the two regions, which is taken as the location of a single or a pair of switching wires, and the split counter-rotating frequencies of a typical resonator is theoretically obtained. This is done by investigating the split cut-off numbers of the related planar circuit with top and bottom electric walls and a magnetic sidewall. The magnetostatic model adopted in this paper assumes equal fluxes in the up and down magnetized regions but unequal flux densities.

*Index Terms*—gyromagnetic resonators, ferrite switches, junction circulators, waveguide

# INTRODUCTION

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he direction of circulation in a circulator is determined by that of the polarity of the direct magnetic field intensity utilized to magnetize its gyromagnetic resonator so that it may be employed to switch an input signal at one port to either one of the other two. One practical means of doing so is to internally or externally latch the resonator by using a current carrying wire loop to switch between the two remanent states of its hysteresis loop. The externally latched device relies on a closed magnetic circuit produced by a post gyromagnetic resonator with top and bottom electric walls and external magnetic elements separated by thin waveguide walls. The internally latched one relies on a wire loop implant within the microwave resonator. The two possibilities differ both in switching speed and in bandwidth. The possibility of switching a junction circulator using an electromagnet is also understood. The switching speed of the electromagnetic activated switch is about 10 msec; that of the externally latched switch is 1 msec and that of the internally latched one about 100 nsec. The bandwidths of the electromagnet and externally actuated devices are of the order of fixed field circulators; that of the internally actuated one is usually less. The literature of this class of switches is given, in chronological order, in [1-11]. The original resonator is a tri-toroidal geometry in the form of a post with top and bottom electric walls latched by a triplet of toroidal return paths each magnetized by a single switching wire. The whole assembly is orientated at 60 degrees from the main waveguides [3]. It approximately produces a uniform magnetization along the central core of the geometry. This concept has been empirically extended with the so called wye resonator replaced by a prism one [4]. The basis of operation of both the original geometries is actually a bit vague, in part due to the fact that the theory of operation of the waveguide circulator was not well understood at the time. An analysis of an internally latched geometry with a single wire embedded in the core of a cylindrical resonator with top and bottom electric walls, under the assumption that both the up and down magnetizations are equally and uniformly magnetized is described in [5,8,9]. A turnstile version using a half-wave long prism resonator with top and bottom magnetic walls is separately dealt with in [10].

The purpose of this paper is to investigate two half-wave long open gyromagnetic cavity resonators used in rectangular waveguide switches, each magnetized by a single wire loop. The main feature of interest in this class of device is the relationship between the splitting of the degenerate counter-rotating modes of each resonator and the magnetic variables. This quantity, to a large extent, fixes the figure of merit of this type of device. Figure la, and b depict the schematic diagrams of the arrangements considered here. Figure 2a shows the details of the wire loop for the cylindrical resonator and 2b that of a half-wave long gyromagnetic cavity consisting of a circular region magnetized in one direction with a triplet of gyromagnetic ribs magnetized in the opposite direction. The optimum geometry is that of the circular gyromagnetic region loaded by a triplet of gyromagnetic ribs. Its split frequencies are essentially unaffected by the magnetic state of the outer ribs of the geometry.

The paper also includes a study of the internal direct magnetization in a cylindrical magnetic insulator using both a single wire loop and a pair of stacked loops.

# Half-Wave Long Gyromagnetic Cavities with Magnetic Walls

An approximate model of the resonator met in connection with the design of the turnstile junction circulator is a half-wave long gyromagnetic waveguide with a magnetic side wall and magnetic end plates. The cross section of a typical waveguide is either circular, triangular or consists of a triplet of ribs attached to a circular region. A fundamental property of

such a cavity, and this is the main calculation of the paper, is its split frequencies. The approach used here involves a calculation based on a perturbation formulation of the split phase constants of the waveguide in terms of the gyrotropy and the split cut-off frequencies of the related planar circuit with top and bottom electric walls and ideal magnetic side walls.



kc is the cut-off number of the isotropic waveguide under consideration, C is a gyrotropy constant that accounts for the degree of splitting between the degenerate counter-rotating modes of the waveguide in question, f is the dielectric constant of the magnetic insulator. This sort of waveguide displays both split propagation constants and split cut-off frequencies.

The constants kc and C are given, in the case of a planar circular resonator with top and bottom electric walls and a magnetic side wall, by





The constants in the case of the waveguide consisting of a circular region to which are attached a triplet of ribs are not available in closed form but can be reduced to polynomial approximations based on existing data [23,24,25,26].





sin and  are defined by





r and R are the radius of the circular region and that of the overall waveguide respectively. w is the width of the ribs. k0 is the usual free space wavenumber. and are the relative diagonal and off diagonal elements of the tensor permeability of the magnetic insulator.  
For a saturated material



 Mr is the remanent magnetization of the garnet material (A/m),  is the gyromagnetic ratio (2.21 x 105 (rad/sec) per (A/m)),  is the free space permeability (4x 10-7 H/m), is the radian frequency (rad/s).

# Planar and Cavity Resonators

The two gyromagnetic resonators met in the description of junction circulators are the planar arrangement and the half-wave long cavity.

The planar resonator with magnetic sidewalls and top and bottom electric walls associated with a typical waveguide under consideration is obtained with



The split cut-off frequencies of the circuit obtained in this way are related to the gyrotropy of the gyromagnetic waveguide by



The boundary condition of the half-wave long cavity with flat magnetic walls 2L apart is



The connection between the split frequencies of such cavities and the constant C is



For the purposes of engineering it is hereafter assumed that the split frequencies of the cut-off space of the planar resonator are representative of those of the cavity. A more accurate description of the resonator including the effect of the image wall is obtained in the usual way by forming a transverse resonance relationship at the open faces of the resonator [16].

The two relationships are equal at the origin for  small. The purpose of this paper is to obtain the constants C for gyromagnetic waveguides with the inner and outer regions magnetized in opposite directions.

The degenerate frequencies of the two resonators are



and



respectively.

# Complex Gyrator Circuit of Junction Circulator Using Half-Wave Cavity with Up and Down Magnetization

The gyrator circuit of any junction circulator for which the in-phase eigen-network may be idealized by a frequency independent short-circuit boundary condition and for which the counter-rotating ones are weakly split by the gyrotropy of the resonator is a 1-port STUB-R circuit [14,15,27]. A knowledge of its element values is a prerequisite for design. This circuit is usually described in terms of its normalized susceptance slope parameter (b’), normalized gyrator conductance (g) and the normalized split frequencies of the degenerate counter-rotating eigenvalues ((+ - )/0). Once the susceptance slope parameter is fixed by the choice of the resonator shape the gyrator conductance is set by the split frequencies of the resonator in the usual way by

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This relationship also defines the quality factor of the circuit as [14,15]



The physical variables of the in-phase eigen-network do not explicitly appear in the description of the complex gyrator circuit provided it may be assumed that it has been idealized by an ideal electric wall at the terminals of the resonator.

Scrutiny of the network problem indicates that values for QL between 0.5 and 2.5 are appropriate for the design of quarter wave coupled devices [28]. An experimental or theoretical knowledge of this latter quantity is therefore essential for design.   
The development of a circulator or switch using an internally latched resonator differs only from that of a conventional one in that the degree of splitting that is realizable between the degenerate counter-rotating modes is in this situation degraded by the up and down magnetization on either side of the wire. This feature may be accounted for by pre-multiplying the gyrotropy constant C in the definition of the split phase constants of the gyromagnetic waveguide by an effective filling factor kfC.

This gives



where   
 

**'**and **'** are the split frequencies of the resonator with its inner region magnetized along the positive z direction and its outer one in the opposite direction. The factor kf may therefore be deduced by evaluating the ratio of the split frequencies of the homogeneous and inhomogeneous circuits respectively.

Taking 0Mr as 0.1200 T (say), the frequency as 4.0 GHz (say), and adopting the upper bound for QL as 2.5 (say) gives the lower bound on the filling factor kf as 0.50.

# Magnetostatic problem in Gyromagnetic Resonators

The symmetry of the problem indicates that the direct magnetic flux in the outer sleeve of the cylindrical resonator, φouter, is equal to that in the inner region, φinner. Furthermore the flux in each of the outer ribs of the tri-toroidal geometries is one third that of the core. The flux density, however, in a typical outer region may be more, or less, than that of the inner. The relationship between the two is



q is the ratio of the surfaces in question.



The ratio of the gyrotropies in the two regions is here assumed to be equal to that of the flux densities. The relationship between the two can be described in terms of the shape factor, q, by



The work assumes a constant gyrotropy κpi in the inner region equal to 0.707 that of the saturated material, 0, which is taken here as 0.7. This means that the gyrotropy in the outer region lies between 0 and 0. When q equals unity the partial gyrotropies in the two regions are equal and opposite and corresponds to the calculation in [8]. The diagonal element of the tensor permeability in this work is taken as unity for simplicity sake. Figure 3 indicates the variation of the gyrotropy in the outer region with the shape factor q, assumed in this work.

The removal of the degeneracy between the counter-rotating modes of the resonator, the main endeavour of this work, is not only dependent on the gyrotropies in the two regions but also on the distribution of the alternating magnetic field within each region. The alternating magnetic field is of course more intense on the axis of the resonator than on its periphery and correctly forms part of the FE calculation.

The magnetostatic problem in the case of the circulator geometry is summarized in figure 4.

The condition in [11] differs from that adopted in [8] when the inner and outer gyrotropies are assumed out of phase but equal in amplitude.

# Cylindrical Cavity with Uniform Up and Down Magnetizations

A closed form characteristic equation governing the split cut-off frequencies of a planar disk resonator with top and bottom electric walls and a magnetic sidewall with a circular switching wire from which those of the related cavity may be calculated is available in the literature [8]. It correctly takes into account the distribution of the alternating magnetic fields within the problem region but, however, assumes that the gyrotropies in the up and down regions are equal. In the model adopted here the gyrotropies are governed by the flux densities in the respective planar surfaces. The gyrotropy in the outer core may therefore be more, or less, than that of the inner one. The ratio of the surfaces magnetized in opposite directions is, in the case of the planar disk resonator illustrated in fig. 4, is given by



Figure 5 illustrates some FE calculations on the split cut-off frequencies in the case of such a planar disk resonator with its inner and outer regions biased in opposite directions. The outer radius of the planar geometry has been chosen to give a demagnetized cut-off frequency of 4 GHz for the purpose of design. The up and down surfaces are equal provided



This situation corresponds to the specific case investigated in [8].

The relationship between the split frequencies and the shape factor of the gyromagnetic resonator is indicated in figure 5 for the situation for which κ0=0.70, κi=0.50. Whereas the magnetic flux density in the outside surface increases with the factor q the corresponding alternating magnetic field drops rapidly there so that the outside surfaces acts as the return path of the gyromagnetic resonator. The corresponding quality factor QL of the resonator is indicated later in the text.

This solution does not display an interchange in the polarization of the counter-rotating split frequencies in the two regions of the resonator unlike the situation noted in (11). This disparity is attributed to the different profiles of the flux densities adopted in the two works.

# The Wye Resonator with Up and Down Magnetization

The cut-off space of the tri-toroidal half-wave long cavity structure composed of a circular region and a triplet of gyromagnetic ribs incorporating a switching wire is the main topic of this paper. Its cut-off space is described, in addition to the location of the wire, by its coupling angle, ψ and its interior radius, k0r. Typical mode charts are available in [24]. The topology under consideration is indicated in figure 6.

The radial wavenumber k0R, or the electrical length, of the ribs or the ratio of the internal and external radius r/R, is the unknown of the problem.

One specific family of solutions is obtained by placing the wires at the ports of the circular gyromagnetic region and adjusting the coupling angle at the same boundary.

The ratio of the surfaces magnetized up and down, for the situation for which the location of the wire coincides with the radius of the circular region of the geometry, is



The up and down magnetized surfaces are equal provided



A separate calculation elsewhere indicates that the opening between the split cut-off frequencies of this sort of circuit deteriorates rapidly for r/R below about 0.50. The up and down problem is therefore restricted to the interval 0.50 ≤ r/R ≤ 1.0.

The physical parameters of the degenerate cut-off space are obtained by varying or k0R using an FE process. One solution at 4.0 GHz is







The unknown of the problem is kR. The result is



The electrical length of the UE or rib is defined in eqn 7.

It is given by



The relationship between the split cut-off frequencies of arrangements with  =0.25 and 0.35 and the shape factor q are indicated in fig. 8. The gyrotropy factor, kf, is separately indicated in figure 9.

The split frequencies of a homogenous and an inhomogeneous magnetized resonator are compared in fig. 10 for one specific value of coupling angle. A scrutiny of this result indicates that the two situations are not very different. This feature may be understood by recognizing that there is no rotation, in the absence of any edge mode effects, in the strips and that the effective permeability is unaffected by the polarity of the magnetization.

These two observations reinforce the interpretation of the arrangement in terms of a tri-toroidal magnetic circuit [11]. One difference between the two arrangements, however, is that the magnetization in the strips, in the up and down situation may be less or more than that of the circular region with a corresponding perturbation of the permeability in the strips and an increase or decrease of the electrical length of the strips compared with that prevailing in the uniformly magnetized overall circuit

This result also suggests that the straight strips of the geometry make no or little contribution to the splitting of the degenerate counter-rotating cut-off frequencies.

# The Optimum Up and Down Magnetized Planar Resonator

The quality factors displayed by the two geometries investigated are compared here. Fig 11 summarizes the result. Taking QL as 2, as a desirable quality factor for the purpose of design, suggests that all four geometries in figure 11 are at first sight equally acceptable. The toriodal geometries, however, come closer to the visualization of the geometry as a tri-toriodal circuit with the direct magnetic fluxes in the return paths one third of that at its core.

# Multiwire Magnetostatic Problem

The boundary between the up and down uniformly magnetized surfaces is, for each of the three resonators considered in this work, the main endeavour of this paper.

One way to explore the internal direct magnetization of the magnetic insulator in the presence of one or more loops is to have recourse to a magnetostatic solver. Fig. 12 shows the magnetizing effect of a single circular wire loop of radius r, carrying 10 A, on a cylindrical resonator with radius R. One feature of this result is that the magnetization on the axis of the loop is inversely proportional to its radius so that such switches are more readily realized at high frequencies than at lower ones. A scrutiny of a typical result indicates that the optimum position of the wire(s) does not necessarily coincide with the situation for which the up and down surfaces are equal. One possible model of such a resonator is one with a demagnetized concentric region, a second with magnetization in one sense, a third with a magnetization in the other sense and an inner circular region with still another value. Such a model can readily be set up, in the case of a cylindrical geometry in closed form or in general using a commercial F.E. package. Fig. 13 depicts a similar result in the case of the pair of stacked circular loops. The spacing between the wires is half the thickness of the resonator.

The inductance of the wire configuration using two wire loops is four times that of the single loop.

The stored energy is likewise increased by a factor of four. This energy, divided by the switching time, is related to the instantaneous power required from the driver.

# Conclusion

One practical resonator in the design of a latched junction circulator using a wire implant is a half-wave long gyromagnetic cavity with either a circular or a circular region with a triplet of rectangular ribs with its flat faces loaded by magnetic or open walls. The gain bandwidth of this type of resonator, when embedded in a degree-2 arrangement is determined by its split cut-off frequencies. The purpose of this paper is to investigate this problem in each of the geometries under consideration. This is done by investigating the related problem of a planar resonator with up and down magnetizations produced on either side of the boundary defined by the switching wire. The model utilized in this development disregards the spatial variations of the magnetization in both the up and down magnetized regions of the magnetic insulator. It provides, therefore, an upper bound of what can be realized in practice. The main conclusion of this work is that the magnetostatic rather than electromagnetic problem is the limiting factor of this sort of problem.

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**Fig 1a Schematic diagram of waveguide junction circulator using a cylindrical resonator with a wire loop.**



**Fig 1b Schematic diagram of waveguide junction circulator using a half-wave long wye resonator with a wire loop.**



**Fig 2a Details of switching wire embedded in half wave long cylindrical resonator.**



**Fig 2b Details of switching wire embedded in half wave long wye resonator.**



**Fig. 3 Uniform partial gyrotropy in inner region of a composite gyromagnetic resonator versus factor q (ratio of up and down surfaces) with κpi=0.5 (κ0=0.7)**



**Fig 4 Uniform up and down magnetizations in a planar resonator with top and bottom electric walls, and a magnetic side wall**



**Fig 5 Split cut-off frequencies of planar disk resonator for constant inner magnetization versus ratio of up and down surfaces. (0=0.7, pi=0.5)**

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**Fig 6 FE discretization of planar wye resonator with up and down magnetization**



**Fig 7 Relationship between coupling angle and R/r for parametric values of shape factor q**



**Fig 8 Split cut-off frequencies of wye resonators with up and down magnetization for parametric values of ψ**



**Fig 9 Ratio of frequency splitting kf versus ratio of up and down magnetised areas (q) for parametric values of coupling angle ψ**

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**Fig 10 Split frequencies of the homogenous and inhomogeneous magnetized resonators versus shape factor ,q, ψ=0.25 and pi=0.5**



**Fig 11 Quality factor of cavity as a function of up and down magnetized surfaces. (0=0.7, pi=0.5)**



**Fig 12 Up and down direct magnetic field strength in cylindrical resonator using single wire loop using a magnetostatic solver. (r/R=0.5, r/R=0.6, r/R=0.707)**



**Fig 13 Up and down direct magnetic field strength in cylindrical resonator using a pair of wire loops using a magnetostatic solver. (r/R=0.5, r/R=0.6, r/R=0.707)**