Split Frequencies and Susceptances of the Three Port Junction Circulator from an Experimental Determination of the Complex Gyrator Circuit.

Joseph Helszajn, John Sharp, and Mark Mckay

Abstract— The adjustment of the classic three port circulator involves an in-phase eigen-network and a pair of split counter-rotating ones. The purpose of this paper is to use some recent relationships to experimentally extract the split susceptances of the counter-rotating immittances and the susceptance slope parameter of the junction by having recourse to a 1-port measurement of its complex gyrator circuit. This is done under the assumption that the in-phase eigen-network of the junction may be idealized by an electric wall. The split frequencies of the counter-rotating eigen-networks of the circulator may also be deduced from this characterization and are in good agreement, under the same assumption, with those derived from 1-port measurements on a terminated circulator. The latter arrangement does not, however, allow the split susceptances of the counter-rotating eigen-networks to be deduced.

*Index Terms*—Circulators, non-reciprocal devices, eigenvalues, characterization, complex gyrator.

# INTRODUCTION

O

ne exact 1-port description of a junction circulator which is applicable at any frequency, is its complex gyrator circuit. It is defined as the input admittance at one typical port of the junction with one of the two remaining ports decoupled from the input port [1]. A knowledge of this quantity is both necessary and sufficient for the synthesis of this class of nonreciprocal device and has been used in the adjustment of the stripline circulator in [2-8]. It allows the gyrator conductance, susceptance slope parameter and quality factor of the junction, which enters into the exact synthesis problem, to be experimentally deduced. It has not however been used to extract the split eigenvalues and the frequencies of the counter-rotating eigen-networks that enter into the description of the operation of the circulator although means of doing so has recently been mentioned [9]. A knowledge of these split quantities allows the frequency responses of any existing representations of the circulator to be produced. The main purpose of this paper is to remedy this situation under the assumption that the in-phase eigen-network may be idealized by an electric wall at the terminals of the junction.

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The approximation that the in-phase mode may be represented by a frequency independent electric wall is in general quite robust. Means of extracting the in-phase eigen-network has been separately dealt with in [9-15] and is further extended here. Another means of extracting information about the complex gyrator circuit at a number of discrete frequencies is to have recourse to 1-port measurements on a terminated circulator [16-20]. This method readily reveals the split frequencies of the counter-rotating eigenvalues but does not reveal the split susceptances. It does however allow a calculation of the susceptance slope parameter in terms of the split frequencies of the gyrator circuit and its gyrator conductance using a standard relationship [16,18]. Some early circulation solutions based on the immittance at port 1 of a terminated circulator are dealt with in [21,22,23]. A scrutiny of the experimental results undertaken here, although not the main result of this paper, suggests that the two procedures are equally good for the circulator under consideration.

The frequency variations of the immittance and scattering eigenvalues have also been directly experimentally deduced in the literature [28-30], but requires that the amplitude and phase of all three entries of the scattering matrix are measured instead of having recourse to a 1-port measurement as articulated here.

# Complex Gyrator Immittance of 3-port Circulator

The complex gyrator immittance of the 3-port junction circulator is a fundamental quantity in the description of this class of device. Its definition is a classic result in the literature and is reproduced here for completeness sake only [12]. It is defined as the input impedance of the junction at port 1 with port 3 decoupled from port 1.

The voltage current relationships of the network in terms of its open-circuit parameters are

 (1)

The required result is

 (4)

The condition at port 2 is then given by

 (5)

This relationship indicates that terminating each port by  in a cyclic manner is sufficient to match the device [1]. Figure 1 illustrates the schematic diagram of this arrangement.



Figure 1. Definition of complex gyrator circuit of a 3-port circulator

The open-circuit parameters are linear combinations of the impedance eigenvalues of the junction in the usual way.

 (6)

 (7)  (8)

where



,and are the 1-port reactance functions displayed by the in-phase and counter-rotating eigen-networks of the junction.

If the frequency variation of the in-phase impedance eigenvalue  may be neglected compared to those of the degenerate or split ones then an especially simple model for this class of device is available. Its realization starts by simplifying the description of the open-circuit parameters by writing

 (9)

It is advantageous, anticipating the topology of the complex gyrator circuit, to proceed in terms of Yin instead of Zin. This readily gives

 (10)

The imaginary and real parts of  are therefore simple linear combinations of the split susceptance eigenvalues of the junction.

One equivalent circuit of the three port junction circulator is therefore a simple 1-port LCR network. This classic result is illustrated in Figure 2. Furthermore, a knowledge of Y+ and Y- are sufficient to describe this class of device.



Figure 2. Complex gyrator circuit (Z0=0)

Figure 3 gives an experimental Smith chart representation of the complex gyrator circuit of one arrangement for parametric values of H0/M0 over the frequency range 1.20-2.50 GHz. Figures 4 and 5 indicate the same data in Cartesian form. The solid lines in these illustrations indicate the best fit on the experimental data.



**0.20**

**0.40**

**H0/M0=0.10**

**0.60**

**1.00**

1.2GHz

2.5GHz

Figure 3. Smith chart representation of complex gyrator circuit of 2.0GHz circulator for different magnetizing fields (Ψ=0.22rad, 2R=25.4mm)



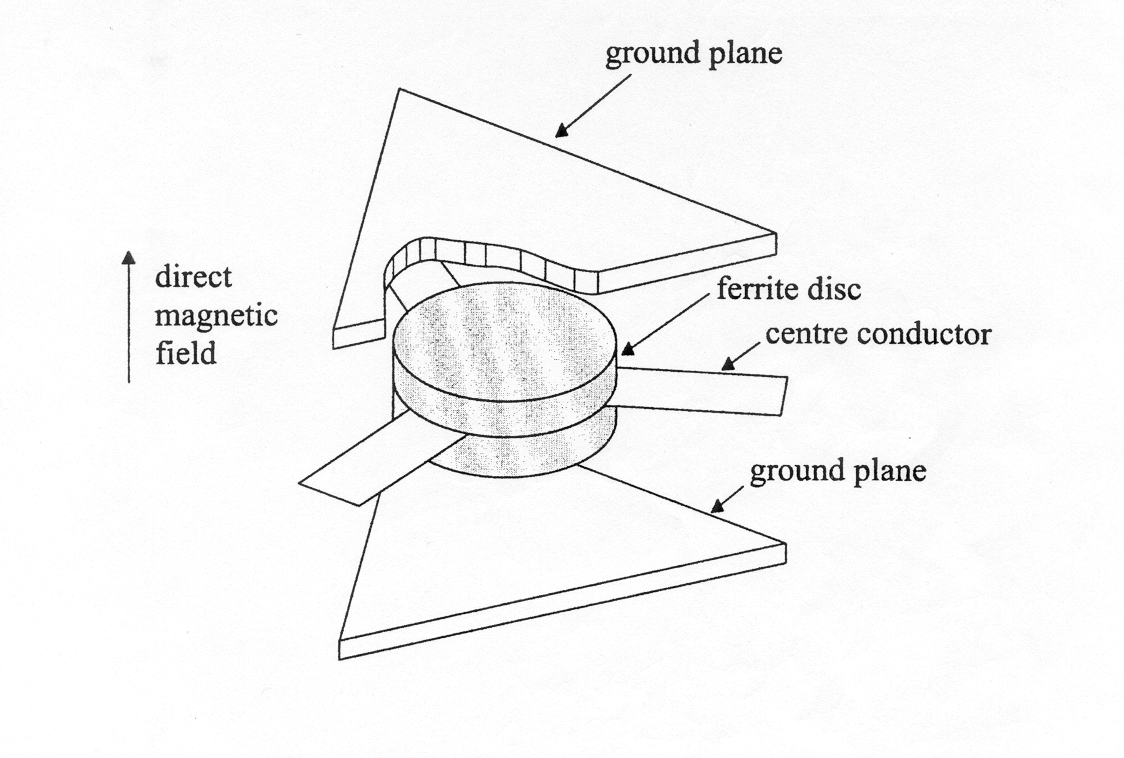


Figure 4. Cartesian representation of normalised gyrator conductance of circulator for different magnetizing fields using Complex Gyrator Method (ψ=0.22 rad, 2R=25.4mm)



Figure 5. Cartesian representation of normalised gyrator susceptance of circulator for different magnetizing fields using Complex Gyrator Method (ψ=0.22rad, 2R=25.4mm)

The experimental arrangement employed in obtaining these results has a frequency response akin to a degree-1 filter network and is often referred to as a degree-1 junction. Its schematic diagram is shown in figure 6. The coupling angle of the ports at the terminal of the junction is ψ=0.22 rad; the radius of the resonator is R=12.7 mm. The thickness of each half-space of the resonator is H=2.0 mm. The garnet material is an Aluminum doped Garnet with a magnetization μ0M0 equal to 0.0400 T and a relative dielectric constant (εf ) of 14.1 .

Figure 6. Topology of a degree-1stripline circulator

The specifications of a gyromagnetic resonator are not complete without a description of the profile of its direct magnetic flux density. For the purposes of simulation the electromagnet coil has been replaced by a permanent magnet in the back rib of the structure. Figure 7 indicates the magnetic flux density at both the position of the probe and through the center of the ferrite for one typical value of H0/M0 using a commercial FE solver. It indicates that the flux density for the physical arrangement employed in this work is essentially uniform across the resonator except for some fringing effect on the edge. The direct magnetic field in the experimental data is taken as that in the air gap of the electromagnetic circuit.



Figure 7. Flux density in gyromagnetic resonator

# Split Frequencies

One means by which the split frequencies of the counter-rotating eigen-networks may be deduced from a knowledge of the complex gyrator circuit has been mentioned in [18,19]. It has not, however, been verified so far. The derivation is repeated here for completeness sake before proceeding with some measurements. It begins by writing the real and imaginary parts of the complex gyrator admittance in normalised form.

 (13)

 (14)

and b are pure imaginary numbers and g is a pure real number.

At , is zero and

 (15)

 (16)

These two equations are compatible provided

 (17)

Likewise at , is zero and

 (18)

The loci associated with these two conditions may be separately placed on a standard Smith chart in the manner indicated in figure 8. The split frequencies of the counter-rotating eigen-network, for a typical degree of gyrotropy, now correspond to the two intersections between the loci and the frequency response of the complex gyrator circuit of the device.



loci

loci

Figure 8. Loci of split frequencies of counter-rotating eigen-networks of the complex gyrator circuit.

Figure 9 compares the split frequencies obtained in this way with those obtained by measuring the frequencies of the 9.5dB return loss points of a terminated circulator. A careful examination of the frequency response of the terminated circulator indicates that there are, above some value of gyrotropy, actually four such frequencies. Figure 10 shows a typical calculated response. The two outside ones have been utilized for this comparison [9].

The complex gyrator circuit procedure outlined here actually avoids ambiguity in the 91/2 dB points.



Figure 9. Split frequencies of circulator versus normalized direct magnetic field (ψ=0.22rad, 2R=25.4mm, f0 = 2GHz) [Complex Gyrator Method (ο); Terminated Circulator Method (Δ)]



**9.5dB circle**

**0.09**

**/=0.06**

**0.12**

**0.15**

**0.20**

-0.20

0.20

Figure 10. Simulated return loss of terminated circulator showing 91/2 dB frequencies (ψ=0.22rad); -0.20 ≤ (f-f0)/f0 ≤ 0.20

# Split susceptances of the 3-port single junction circulator

The possibility of extracting the split eigen-networks from a statement of the complex gyrator circuit has again also been mentioned in the literature although experimental data has not been obtained so far. The purpose of this section is to recapitulate the same and present some data on these quantities. The two conditions are readily obtained by solving equations (15) and (16) for y+ and y-. The results are

 (19)

 (20)

Figure 11 illustrates the result for one value of H0/M0 in the case of the 2.0 GHz arrangement employed in this work.

The susceptance slope parameters at the midband of the junction, and at the split frequencies, may also be evaluated without ado from this sort of diagram. The susceptance slope parameter of the device is a simple linear combination of the split quantities. Figure 12 indicates some data for another value of H0/M0.



Figure 11. Normalised split susceptances of circulator using Complex Gyrator Method for H0/M0=0.10 (ψ=0.22rad, 2R=25.4mm)

The normalized susceptance slope parameter, b´, of the complex gyrator circuit in the vicinity of the midband frequency , is separately obtained by forming

 (21)

The split normalized quantities are obtained from similar relationships. The connection between the susceptance slope parameters obtained in this way and the direct magnetic field intensity are deduced by taking the slopes of the midband susceptances at 0 and ±. These results are indicated in Figure 13.



Figure 12. Normalised split susceptances of circulator using Complex Gyrator Method for H0/M0=0.40 (ψ=0.22rad, 2R=25.4mm)



Figure 13. Normalised split susceptance slope parameters (ψ=0.22rad, 2R=25.4mm)

The quality factor of the complex gyrator circuit may be calculated in terms of b’ and g without ado.

 (22)

# Terminated Circulator

g, b´ and QL may also be deduced by measurements at port 1 with both output ports terminated in matched loads.. The experimental procedure connected with the terminated circulator is a classic result in the literature [16-18]. It does not however permit the extraction of the split counter-rotating susceptances and susceptance slope parameters. The conductance at port 1 of the terminated circulator is not to be confused with that of the gyrator circuit. Figure 14 indicates the agreement between the two different processes in the case of the gyrator conductance. The agreement between the two is excellent. Figure 15 compares the susceptance slope parameters of the two processes. The quality factors based on each experimental procedure are separately compared in figure 16.



Figure 14. Normalised gyrator conductance of circulator versus normalised direct magnetic field at midband frequency (ψ=0.22rad, 2R=25.4mm, f0 = 2GHz) [Complex Gyrator Method (ο); Terminated Circulator Method (Δ)]

# The Synthesis Problem

One task of the paper is to characterize the complex gyrator circuit of a typical stripline junction using a disk gyromagnetic resonator. A knowledge of the gyrator conductance, susceptance slope parameter and quality factor is both necessary and sufficient in order to fix the gain-bandwidth of the circuit once the degree of the matching network is specified. This problem is well understood and is also included for completeness sake [28,29,30].

The complex gyrator circuit measured in this work at is typically specified by





Figure 15. Normalised susceptance slope parameter of circulator versus normalised direct magnetic field at midband frequency (ψ=0.22rad, 2R=25.4mm, f0 = 2GHz) [Complex Gyrator Method (ο); Terminated Circulator Method (Δ)]



Figure 16. Quality factor of circulator versus normalised direct magnetic field (ψ=0.22rad, 2R=25.4mm, f0 = 2GHz) [Complex Gyrator Method (ο); Terminated Circulator Method (Δ)]

A degree-2 frequency response which is compatible with this complex gyrator circuit is



 is the maximum voltage standing wave ratio of the frequency response,  is the minimum value and 20 is the normalised bandwidth. The above specification is compatible with many commercial requirements and is in keeping with practice.

# In-Phase Eigen-Network

The assumptions throughout this work is that the in-phase eigen-network of the circulator may be idealized by an ideal electric wall at the terminals of the junction and that the counter-rotating eigen-networks establish magnetic walls everywhere except over the ports of the junction.

If the in-phase eigen-network cannot be neglected then it may be shown that [13]

 (22)

where

 (23)

and

 (24)

This immittance can be realised as a series impedance in terms of the impedance of the in-phase eigen-network in cascade with a shunt circuit involving simple linear combinations of the counter-rotating split admittances. It is reproduced in figure 17.



Figure 17. Complex gyrator circuit (Z0≠0)

The series element  may be extracted experimentally from the frequency response of the complex gyrator admittance in figure 3. The normalized in-phase eigenvalue obtained in this way is . This quantity is obtained by equalizing the locus of the frequency response of the gyrator impedance about the midband frequency.

The extent that the in-phase eigen-network displays an electric wall at the terminals of the junction may be separately tested by extracting its impedance eigenvalue there. This quantity is related to the in-phase poles of the junction



Figure 18 indicates one typical calculation in the case of a so-called weakly magnetized device. The in-phase poles used in the construction of the corresponding eigenvalue are shown in figure 18.



Figure 18. Normalised in-phase poles of junction circulator (calculation), (ψ=0.22rad, κ/μ=0.25, kR=1.0953)

This sort of plot is of course dependent upon both the gyrotropy and the coupling angle defined by the strips at the terminals of the junction. The value of the gyrotropy utilized in this calculation is equal to 0.25 and the coupling angle is 0.22 rad.

# Conclusion

The split susceptances and split susceptance slope parameters of the counter-rotating eigen-networks of the 3-port junction circulator have, in this paper, been evaluated for the first time from a measurement of its complex gyrator circuit. This has been done under the assumption that the in-phase eigen-network may be idealized by an ideal electric wall. The paper has also compared the midband elements of the complex gyrator circuit based on a direct evaluation of its complex gyrator circuit with those based on the more simple 1-port measurements of a terminated junction. The two procedures are in good agreement.

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