

# PN code acquisition with a CA-CFAR adaptive digital matched filter and its realisation using FPGA

B.Wei, MY. Sharif, AEA.Almaini, TD.Binnie  
School of Engineering  
Napier University

10 Colinton Road, Edinburgh, EH10 5DT, UK  
Phone: (44) 0131-455 2284 Fax: (44) 0131-455 2264 E-mail: b.wei@napier.ac.uk

**Keywords:** Constant False Alarm Rate, Matched Filter, Mean Acquisition Time, FPGA

**Abstract -** In this paper the performance of a Cell Averaging Constant False Alarm Rate (CA-CFAR) Pseudo-Noise (PN) code adaptive detector is analysed for a single path communication channel. The detection process uses a digital Matched Filter (MF) and is implemented using Field Programmable Gate Array (FPGA) technology. The performance of the CA-CFAR detector is compared with Order Statistics CFAR (OS-CFAR), OR CFAR (OR-CFAR) and AND CFAR (AND-CFAR) detectors in terms of speed of acquisition and hardware requirement. Simulation results using Xilinx devices are presented confirming that the CA-CFAR adaptive PN code detector has a rapid acquisition speed (second best after AND-CFAR) and has the least implementation complexity compared to other three detectors.

## 1. INTRODUCTION

In spread spectrum communication receiver systems, to despread the received signal correctly, the transmitter and receiver must be synchronised. Code acquisition is the first step of code synchronisation, which normally aligns the codes with accuracy less than a chip duration. The second step in synchronisation is tracking, which further aligns and maintains the codes to zero phase difference. A digital MF is commonly used in PN code acquisition. It is faster than the correlator in terms of Mean Acquisition Time (MAT), because it produces outputs at each chip time  $T_C$ , while the correlator produces outputs at each dwell time  $T_D$ .

In spread spectrum communication, noise in the channel may lead to bad performance in PN code acquisition. This effect is even worse for non-constant Signal-to-Noise Ratio (SNR), when fixed threshold is used in detection. CFAR detectors can be used in PN code acquisition to minimise the problem caused by noise [1-2].

The merit of using FPGA to implement the code acquisition process is that it can provide a massive parallel structure which is needed in the pipelines both in the MF and the CFAR detector.

## 2. STRUCTURE OF CFAR DETECTORS

Many kinds of CFAR techniques have been developed in order to obtain better performance in noisy environment in mobile communications. The two commonly used are CA-CFAR and OS-CFAR detectors; also, some new

CFAR techniques are developed, like AND-CFAR and OR-CFAR [3].

The difference between CA-CFAR and OS-CFAR detectors is in the method of obtaining the adaptive threshold. By using the adaptive threshold, the probability of false alarm rate  $P_{FA}$  can be made constant. The adaptive threshold is calculated by multiplying a fixed scaling factor,  $T$  and an adaptive value,  $Z$  from the reference cells. For CA-CFAR, the adaptive value,  $Z_{CA}$  is the average of  $N$  cells on the two sides of the cell tested, and for the  $K_{th}$  order OS-CFAR, the adaptive value  $Z_{OS}$  is the  $K_{th}$  biggest sample of  $N$  cells. Figure 1 shows the block diagram of CA-CFAR and OS-CFAR detectors.

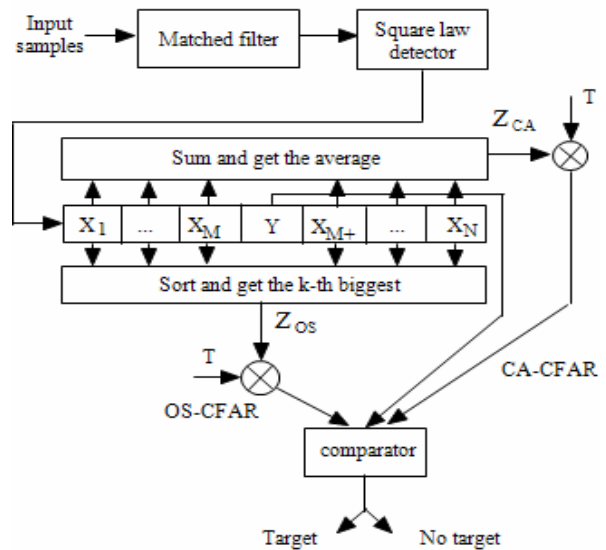


Figure 1. Block diagram of CA-CFAR and OS-CFAR detectors.

The AND-CFAR and OR-CFAR are developed from CA-CFAR and OS-CFAR. For AND-CFAR, when the output  $Y$  is greater than both thresholds, there is a target detection. For OR-CFAR,  $Y$  needs to be bigger than any of the thresholds for target detection.

Equation (1) represents the detection decision function, and equation (2) is for OR-CFAR detector, but  $T$  in these two equations is not equal to that of CA-CFAR or OS-CFAR.

$$(Y > Z_{CA} \times T) \text{ and } (Y > Z_{OS} \times T) \quad (1)$$

$$(Y > Z_{CA} \times T) \text{ or } (Y > Z_{OS} \times T) \quad (2)$$

The AND-CFAR and OR-CFAR have their own scaling factors T which can be calculated by equations (7) and (9).

### 3. ALGORITHMS FOR CFAR DETECTORS

The adaptive threshold for CA-CFAR and OS-CFAR detectors can be calculated as follows [4]. For CA-CFAR and OS-CFAR detectors the probability of false alarm  $P_{FA}$  can be obtained from:

$$P_{FA}(CA) = \frac{1}{(1 + T/N)^N} \quad (3)$$

and

$$P_{FA}(OS) = k \binom{N}{K} \frac{\Gamma(N - K + T + 1)\Gamma(K)}{\Gamma(N + T + 1)} \quad (4)$$

respectively.

$P_{FA}$  is the constant false alarm rate and N is the number of reference cells used. In equation (3) N and  $P_{FA}$  are known, thus T can be obtained easily.

In equation (4), there is one more parameter K, which means the  $K_{th}$  biggest sample from the reference cells is used. When  $P_{FA}$ , N and K are known, T can be calculated.

For CA-CFAR and OS-CFAR detectors the probability of detection  $P_D$  can be obtained from [4]:

$$P_D(CA) = \frac{1}{\{1 + T / [(1 + \lambda)N]\}^N} \quad (5)$$

and

$$P_D(OS) = k \binom{N}{K} \frac{\Gamma[N - K + T / (1 + \lambda) + 1]\Gamma(K)}{\Gamma[N + T / (1 + \lambda) + 1]} \quad (6)$$

respectively.

In equations (5) and (6)  $\lambda$  denotes the SNR. By using equation (3) and (4), the scaling factor T can be obtained and therefore  $P_D$  can be calculated.

$P_{FA}$  and  $P_D$  for AND-CFAR and OR-CFAR detectors can be obtained as follows [4]. For AND-CFAR detector:

$$P_{FA}(AND) = k \binom{N}{K} \left\{ \frac{\Gamma[N - K + T / (1 + \lambda) + 1]\Gamma(K)}{\Gamma[N + T / (1 + \lambda) + 1]} - \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{K-1} \binom{K-1}{j} (-1)^j \left[ \frac{N}{2N - K + j + T / (1 + \lambda) + 1} \right]^{j+1} \right\} + \sum_{i=K}^N \binom{N}{i} \sum_{j=0}^i \binom{i}{j} (-1)^j \left[ \frac{N}{2N + j - i + T / (1 + \lambda)} \right]^N \quad (7)$$

$$P_D(AND) = k \binom{N}{K} \left[ \frac{\Gamma(N - K + T + 1)\Gamma(K)}{\Gamma(N + T + 1)} - \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{K-1} \binom{K-1}{j} (-1)^j \left( \frac{N}{2N - K + j + T + 1} \right)^{j+1} \right] + \sum_{i=K}^N \binom{N}{i} \sum_{j=0}^i \binom{i}{j} (-1)^j \left( \frac{N}{2N + j - i + T} \right)^N \quad (8)$$

For OR-CFAR detector:

$$P_{FA}(OR) = \frac{1}{(1 + T/N)^N} - \frac{1}{N} K \sum_{i=0}^{N-1} \sum_{j=0}^{K-1} \binom{K-1}{j} (-1)^j \left( \frac{N}{2N - K + j + T + 1} \right)^{j+1} - \sum_{i=K}^N \binom{N}{i} \sum_{j=0}^i \binom{i}{j} (-1)^j \left( \frac{N}{2N + j - i + T} \right)^N \quad (9)$$

$$P_D(OR) = \frac{1}{[1 + T / (1 + \lambda) / N]^N} - \frac{1}{N} K \sum_{i=0}^{N-1} \sum_{j=0}^{K-1} \binom{K-1}{j} (-1)^j \left[ \frac{N}{2N - K + j + T / (1 + \lambda) + 1} \right]^{j+1} - \sum_{i=K}^N \binom{N}{i} \sum_{j=0}^i \binom{i}{j} (-1)^j \left( \frac{N}{2N + j - i + T / (1 + \lambda)} \right)^N \quad (10)$$

For AND-CFAR and OR-CFAR, when  $P_{FA}$  is set to be a constant, knowing K and  $\lambda$ , T and  $P_D$  can also be calculated.

MAT is an important factor to evaluate the performance of PN code acquisition, because the speed of acquisition is very important. When  $P_D$  and  $P_{FA}$  are known, MAT can be calculated by using the following equation:

$$MAT = \sum_{0}^{N=\infty} \{ [(N + 0.5) \times L + P_{FA} / (1 - P_{FA})] \times P_D \times (1 - P_D)^{N-1} \times T_C \} \quad (11)$$

Where L is the length of PN code, and  $T_C$  is the chip duration of the PN code. When there is no "false" and "miss" in detection, the average acquisition time should be  $1.5 \times L \times T_C$ . A "miss" will delay the detection at least L chips and a false detection will only delay the acquisition by 1 chip. Accumulation of the detection time multiplied by the probability, at which it happens, yields the MAT of the detection.

#### 4. PERFORMANCE ANALYSIS FOR THE FOUR CFAR DETECTORS

The following simulations have been carried out using MATLAB software development tools.

Figure 2 shows the simulation results for different numbers of reference cells,  $N$ , to be used in CFAR detectors. It can be seen that a bigger  $N$  gives a better  $P_D$ . However, the improvement is not constant for the same increase in the number of cells and it decreases as the number of cells is increased. So, in a practical application, the minimum  $N$  that meets the required performance can be determined and this would be the most economical value.

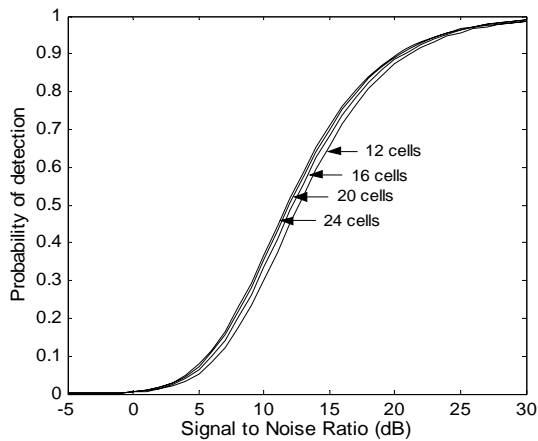


Figure 2. Probability of detection for CA-CFAR detectors with different  $N$ .  $P_{FA} = 0.0001$ .

In this paper, for all simulations the following parameters have been used:  $N = 16$ ,  $P_{FA} = 0.0001$ , and  $K = 13$  and  $11$  to see how the performance of OS-CFAR, OR-CFAR and AND-CFAR changes when  $K$  is different.

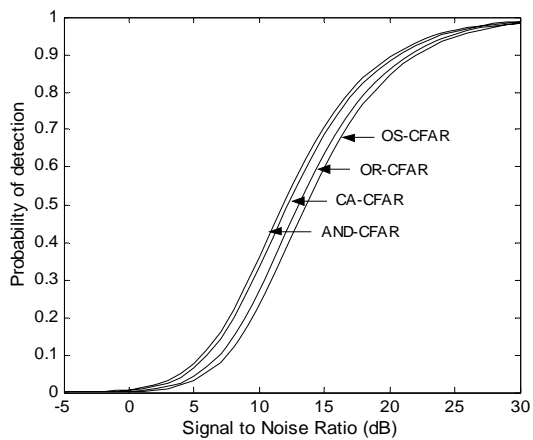


Figure 3. Probability of detection of four CFAR detectors.  $P_{FA} = 0.0001$ ,  $N = 16$  and  $K = 11$ .

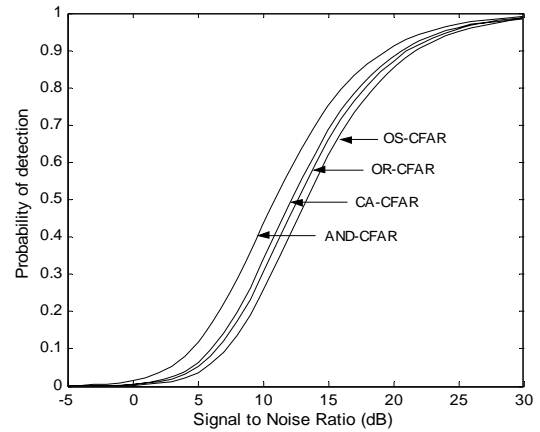


Figure 4. Probability of detection of four CFAR detectors.  $P_{FA} = 0.0001$ ,  $N = 16$  and  $K = 13$ .

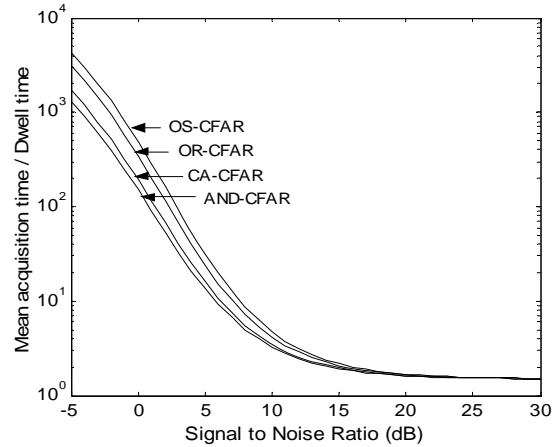


Figure 5. Mean acquisition time for four CFAR detectors.  $N=16$ ,  $K=11$ ,  $P_{FA}=0.0001$ .

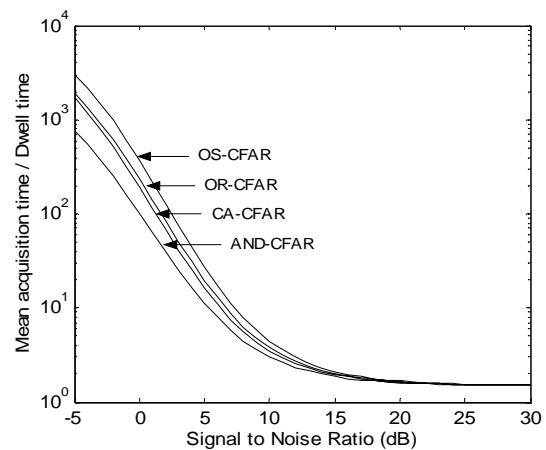


Figure 6. Mean acquisition time for four CFAR detectors.  $N=16$ ,  $K=13$ ,  $P_{FA}=0.0001$ .

From the simulation results presented in figures 3 to 6, it can be seen that in a single path communication channel, an AND-CFAR detector always gives the best performance, in terms of  $P_D$  and MAT, followed by CA-

CFAR, OR-CFAR and OS-CFAR respectively. When the parameter  $K$  is decreased, the performance of AND-CFAR, OR-CFAR and OS-CFAR is degraded. However, the performance of the AND-CFAR detector is always the best, especially with a large value of  $K$ .

For the “cell averaging” process, the accumulator works as in the following equation:

$$Z = Z + X_I - X_O \quad (12)$$

Where  $Z$  is the accumulator output,  $Z'$  is the value of the previous accumulation,  $X_I$  is the new input data to the reference cells, and  $X_O$  is the discarded sample from the shift register. The “cell ordering” process is much more complex compared with the “cell averaging” process; with one comparator the average computation to find a  $K_{th}$  order cell is  $(N-1) \times N/2$  (comparing one cell with the others requires  $N-1$  computations, and the average computational load is equal to the computation for the cell in the middle). AND-CFAR and OR-CFAR involve both “cell averaging” and “cell ordering” computations. This will lead to unsynchronised  $Z_{CA}$  and  $Z_{OS}$ , and it is difficult to deal with this problem. So, in this paper, a CA-CFAR detector is chosen to produce the digital MF adaptive function.

## 5. REALISATION OF CA-CFAR DIGITAL MATCHED FILTER USING FPGA

### 5.1 Structure of the circuit

A 256-coefficient MF with a CA-CFAR detector is presented in this paper. The MF part of the circuit can be designed with the pipeline structure. Figure 7 shows the structure of a MF with transposed Finite Impulse Response (FIR) structure. The merit of using a transposed FIR structure is the lower input to output latency [5].

The main structure of the CA-CFAR detector is a shift register. For a MF with 256 coefficients and 8-bit input, the output will be 16 bits, considering the worst case situation. Thus, the proposed input of the CA-CFAR detector is 16 bits and with 16 reference cells used, the register needs to be 16 bits in width and 17 bits in length (16 reference cells and 1 cell under test). The rest of the CA-CFAR detector includes an accumulator, a multiplier and a comparator, all in 20 bits.

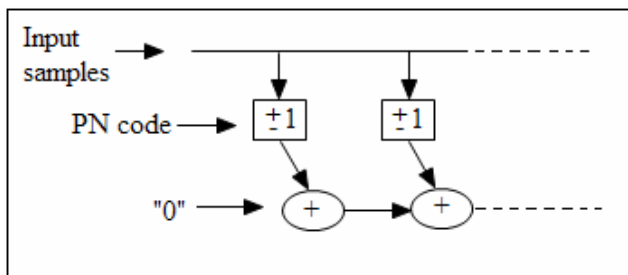


Figure 7. Matched filter with transposed FIR structure

### 5.2 Simulations with VHDL

ISE from the Xilinx is used as a design tool in the simulations, and “Virtex-E XCV600E” is chosen as the target device.

Firstly, the MF part and CA-CFAR detector part are coded with Very High Speed Integrated Hardware Description Language (VHDL), and then Modelsim is used as the simulator for the pre-routed and post-routed design to verify the logic function and timing.

### 5.3 Layout with FPGA

After simulations, the circuit of MF with CA-CFAR detector is successfully implemented on the “Virtex-E XCV600E”. The slices consumed are 2236 out of the total 6912 in the device. It is about 32.3% of the whole device and the total delay is 4.799 ns.

## 6. CONCLUSION

Four types of CFAR techniques are analysed in this paper and the results show that in a single path communication channel, an AND-CFAR detector has much better  $P_D$  and thus shorter MAT. The simulation results confirmed that the CA-CFAR detector has the second best performance, after AND-CFAR. However, its implementation complexity is much less than the other three detectors. The FPGA hardware implementation of a MF with CA-CFAR detector is easily achieved with the device “Virtex-E XCV600E” of Xilinx, with only 32.3% of the whole slices used.

When a multipath situation exists in communication, a CA-CFAR detector will not give a good performance as discussed in [6]. Signal transmitted from different paths may exist in the  $N$  reference cells at the same time, and the “cell averaging” gives bigger values of  $Z_{CA}$ . This leads to a bigger threshold and consequently much more “miss” in detection. Currently research is being carried out to investigate the performance of adaptive PN code acquisition in multipath channel.

## REFERENCES

- [1] R. L. Pickholtz, D. L. Schilling, and L. B. Milstein, “Theory of spread-spectrum communications-A tutorial,” *IEEE Trans. Communication*, vol. 30, pp. 855-884, 1982
- [2] A. J. Viterbi, “CDMA: Principles of Spread Spectrum Communication”, reading, MA: Addison-Wesley, 1995
- [3] P. P. Gandhi and S. A. Kassam, “Analysis of CFAR processors in non-homogeneous background”, *IEEE Trans. Aerosp, Electr Systems*, vol.24, pp. 427-445, 1988
- [4] L. Zhao, W. X. Liu, X. Wu and J. S. Fu, “ A Novel Approach for CFAR Processors Design”, *IEEE Trans. Oceanic Engineering*, vol.30, pp.900-911, 2005
- [5] K. Chapman, P. Hardy, A. Miller, M. George, “CDMA Matched Filter Implementation in Virtex Devices”, *Xilinx application notes*, XAPP212, 2001
- [6] C. J. Kim, H. J. Lee, H. S. Lee, “Adaptive Acquisition of PN Sequences for DSSS communication”, *IEEE Trans. Communication*, vol. 46, pp. 993-996, 1998