A Symbolic Investigation of Pole-Zero Cancellation for a Triple Inverted Pendulum

C. A. Woodham, H. Su and P.J. Larcombe

School of Mathematics and Computing

University of Derby

Kedleston Road, Derby, DE22 1GB, U.K.

Abstract

The balancing of an inverted pendulum is a classic control problem of some thirty years or so standing, which has been revisited of late in the light of recent developments in symbolic computation [1,2]. The controllability of the damped double inverted pendulum has previously been investigated with the aid of symbolic computation, and some interesting results have been obtained. In particular, the transfer functions with respect to the possible control inputs, for the three balancing states, have been calculated symbolically, and the cancelling poles have been extracted in full algebraic form.

The more complex problem of the triple inverted pendulum [3], which has, not a curve of uncontrollability (with respect to the force on the trolley), but a surface of uncontrollability, has also been considered. Numerical and symbolic investigations have already been completed for the cases when only one of the three damping coefficients is non-zero, for the quasi-uniform system - i.e. when the masses of the arms and the trolley are set equal, and the lengths of the arms are set equal [4]. This simplification is necessary to enable the symbolic investigation to be undertaken. In this paper we address the case when two out of the three damping coefficients are non-zero, and also the case when all three damping coefficients are non-zero, but assumed equal to each other. The increased complexity of this problem has necessitated an exploratory numerical investigation of the transfer functions. The numerical results for the cancelling pole, along with some deductions concerning its symbolic form, have facilitated a symbolic investigation.

References

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