Examination of saturated hydraulic conductivity using grading curve functions

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ABSTRACT: In a previous research program 74 artificial soil mixtures of natural fluvial soils were prepared in 4 series of measurements for falling head permeability testing, differing in d_{10}. The conclusions drawn from the original investigations were as follows: the k showed a decreasing tendency with the increase of the uniformity index U for each series of measurements and increasing tendency with d_{10}. In this research these are used (i) to test the value of the grading entropy parameters in case of non-precise grading curve measurement with missing fines, (ii) to correlate the usual grading curve parameters like d_{10}, d_{30}, d_{50} and d_{60} or their ratios and the grading entropy parameters, (iii) to validate some existing permeability – grading curve equations and to elaborate some new permeability – grading curve relationships partly with entropy parameters partly with the usual parameters. For these aims, series I to IV have been started to be reevaluated, some specific surface formulae were derived. Some additional, literature data were also considered. The very first results are presented here. According to the results, the fine fractions significantly influenced the value of the entropy parameters. The base entropy S_o showed strong relationship with and parameters like d_{10}, d_{30}, d_{50} and d_{60}. The entropy increment ΔS showed a monotonic increasing relationship with U. The specific surface parameter (containing density info) showed the best relationship with k out of the d-type parameters like d_{10} or harmonic mean d_h. The original conclusions were reformulated in terms of S_0 and and ΔS. It was also concluded that those parameters that are based on all measured data are more precise than the single diameter values in the k-regressions. The specific surface parameter is the best in this repect probably since containing density information, too.

Keywords: Grading Entropy; Hydraulic Conductivity; Grading Curve Functions; Void Ratio, Specific surface area

1. Introduction

The measured grading curve is an empirical distribution function (i.e., the cumulative distribution function is a step function which can be considered as a discrete distribution function without loss of information). The empirical central moments or statistical entropy can be used to characterize this empirical distribution function since these parameters contain all measured data in a consistent way.

At present approximate, quantile-type statistical quantities are used. In addition, aome sources of error is related to the grading curve measurement (e.g., neglecting the small fractions). Using approximatethe variables, it is difficult to elaborate empirical relationships between soil parameters and the grading curve ([1 to 12].

In this paper, a tentative attempt is made to proper incorporate the grain size distribution and its evolution on the permeability function of granular material. The data of some previous research projects [10] are used (i) to test the value of the grading entropy parameters in case of non-precise grading curve measurement with missing fines, (ii) to correlate the usual grading curve parameters and the grading entropy parameters, (iii) to validate some permeability – grading curve equations.

Using the above procedure, the values of k obtained in the framework of Nagy project [10] are plotted in the entropy diagrams and the results are compared with the results reported by Feng et al. [1,2].

The results are used to represent the permeability values in the non-normalised grading entropy diagram by level lines.

2. Literature review

2.1. Grading curve

2.1.1. General approach

In order to classify a soil on the basis of its particle sizes, it is necessary to quantify the sizes of the particles present in a soil. For particles down to a size of around 60-75 microns (0.06-0.075mm), this can be done using wet or dry sieving. Particles below this size are too small to be separated using sieves, and so, other techniques are needed. Large particles will fall freely under gravity. For small particles, electrostatic attractions begin become significant, and can dominate the particle's behaviour. Consequently, when sieving fine soils, it is usual that individual dry particles will clump of agglomerate together to form apparently larger particles that will not disaggregate and pass through a sieve [12].

Although free small particles tend to remain free when suspended in water, the residual effects of sedimentation often prevent small particles from disaggregating when the soil is first wetted. To promote particle disaggregation, a chemical dispersant is usually added to the water. The dispersant which is a mixture of sodium hexametaphosphate and sodium carbonate, helps to overcome the near surface attractions between clay particles and the cementation between clays and other fine particles.

There are a number of techniques available to estimate the sizes of finer particles in a soil. The most common are the hydrometer analysis, and laser diffraction techniques. Hydrometer analysis is an indirect method that is relatively slow and difficult to perform.

The principles behind the hydrometer analysis are that larger particles settle from a suspension before smaller ones, and the amount of different sizes in a given suspension can be inferred by the rate of change of density of the suspension, by monitoring the height at which an object (hydrometer) floats. A modern alternative to hydrometer analysis is the laser diffraction sizing method. By analyzing the diffraction patterns produced when laser beams are shone through a suspension of fine particles, it is theoretically possible to discriminate particles as small as 0.01 microns.

Both hydrometer analyses and laser diffraction analyses rely on the assumption of spherical particles. In soils with a high clay content, where the sheet-like clay particles may be 100 times wider than they are thick, the errors can be considerable.

However, if the potential errors are addressed and/or appreciated, both hydrometer analyses and laser diffraction analyses can give results with acceptable accuracy.

2.1.2. Statistical characterization

Once the relative proportions of the different grain size fractions (expressed as percentages by total mass of the soil), have been determined, they are usually presented on a Particle Size Distribution (PSD) curve, shown in Fig. 1.

From the PSD, two quantities are defined in terms of the basic particle size distribution curve data. These are the coefficient of uniformity C_u, given by $C_u = \frac{D_{60}}{D_{10}}$ and the C_c. A granular soil is considered well graded if C_u >4 and 1< C_c< 3, otherwise it is poorly graded.

In the case of the empirical grading curve, the central moments or the statistical entropy have not been used in the practice. Instead of it, the q-quantiles like like d_{10} are determined approximately.

2.2. Statistical viewpoints

2.2.1. Statistical cumulative distribution function

The measured grading curve can be considered as an empirical distribution function. In statistics, an empirical distribution function is the distribution function associated with the empirical measure of a sample, a step function, a discrete distribution function.

The expected value is the mass center of a distribution, the variance measures how far a set of (random) numbers are spread out from their expected or average value. The skewness is a measure of the asymmetry of the probability distribution, kurtosis is a descriptor of the shape of a probability distribution. Using these, the type of the distribution can be estimated.

The expexted μ of the random variable of a *discrete* distribution is the weighted arithmetic mean of the possible values (x_1,..., x_k) of ξ

$$\mu = M(\xi) = \sum_k x_k p_k \tag{1}$$

The variance of a *discrete* variable may be determined by the following expression

$$D(\xi) = \sum_i \left[x_i - M(\xi)\right]^2 pi = \sum_i x^2 pi - M^2(\xi) \tag{2}$$

The coefficient of skewness (C_s) is the quotient of the third central moment of the standard deviation raised to the third power.

$$C_s = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^3 p_i}{D^3} \tag{3}$$

The kurtosis(C_k) is the quotient of the fourth central moment by the fourth power of the standard deviation minus 3.

$$C_s = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4 p_i}{D^4} - 3 \tag{4}$$

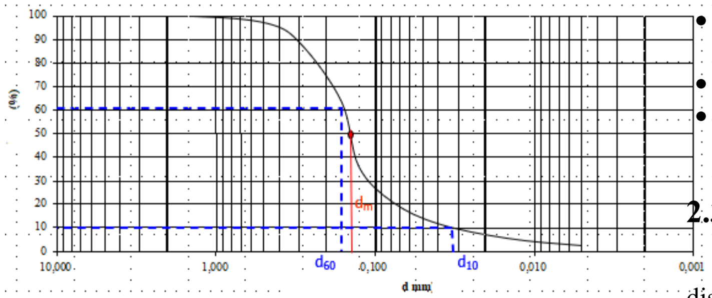
Réthái [3] and Elderton and Johnson [4] distinguishes seven types of curves (I-VII), depending on the value of the so-called criterion, defined by the expression

$$K = \frac{\beta_1(\beta_2 + 3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)} \tag{5}$$

$$\beta_1 = C_s^2 \tag{6a}$$

$$\beta_2 = C_k + 3 \tag{6b}$$

where β_1 is characterizing the skewness and β_2 is characterizing the kurtosis or peakiness.



(a)

(b)

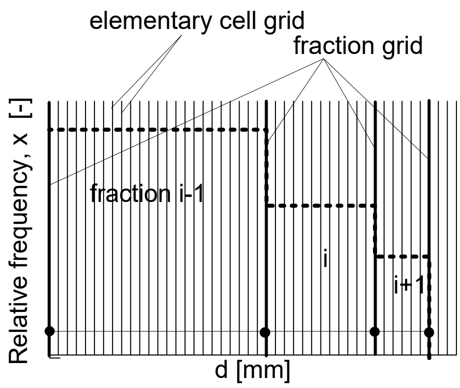


Figure 1. (a) Some grading curve parameters used in practice. (b) The grading density curve embedded in the elementary cell grid, assuming uniform distribution within the fractions.

Table 1. Definition and properties of fraction *j*.

j	1	23	24
Limits in d_0	1 to 2	2^{22} to 2^{23}	2^{23} to 2^{24}
D or S_{0i} [-]	1	23	24

2.2.2. Statistical entropy

The statistical entropy (the entropy of a distribution function) is presented in many textbooks and can be formulated as follows in the discrete case. Let us consider M elements in m equal cells, M_i is the number of the elements in the i-th cell. The statistical entropy S_s:

$$S_s = M_s \tag{7}$$

where s is the specific entropy or the entropy of an element given by

$$s = -\sum_{i=1}^{m} \alpha_i \log_b \alpha_i \tag{8}$$

In equation 2, b is the base of the logarithm, and α_i is the relative frequency of the i-th cell, given by

$$\alpha_i = \frac{M_i}{M} \tag{9}$$

2.3. The grading entropy

Grading entropy [5] has been proved useful in the following fields:

- rules for granular filters, particle migration and segregation criteria [6, 7],
- dry density of granular soils in the loosest possible ("e_{max}") state [8],
- dispersive and piping nature of soils [9],
- relation between grading curve and soil water characteristic curve [10], *k*.

2.3.1. Parameters

For the statistical entropy of a finite discrete distribution, a uniform cell system is used (Fig. 2.). In the case of the empirical grading curve, the base of the logarithm is set to 2 in the statistical entropy formula:

$$s = -\frac{1}{\ln 2}\sum_{i=1}^{m} \alpha_i \ln \alpha_i \tag{10}$$

so that the maximal value of the specific entropy of a two-cell system could be equal to 1 where the relative frequencies of a two cells are equal.

The empirical grain size distribution curve is a finite discrete distribution. The statistical entropy is computed using two statistical cell systems. The so called fractions - which are measured - are defined by successive multiplication with a factor of 2, starting from an arbitrary d_0 as follows (j =1, 2.., Table 1).

$$2^j d_0 \geq d > 2^{j-1} d_0 \tag{11}$$

where fractions are numbered by j (serial number).

The relative frequencies x_i can be identified with the barycentre coordinates of the points of an N-1 dimensional, closed simplex (which is the N-1 dimensional analogy of the triangle or tetrahedron, the 2 and 3 dimensional instances) and, the space of the grading curves with N fractions can be identified with the N-1 dimensional, closed simplex.

The elementary cells are with d_0 width assuming that the distribution within a fraction is uniform. The "smallest diameter" d_o may be taken arbitrarily, eg., to be equal to the height of SiO_4 tetrahedron (d_o=2^{-29} m).

The number of the elementary cells C_i in the fraction i is equal to:

$$C_i = \frac{2^i d_0 - 2^{i-1} d_0}{d_0} = 2^{i-1} \tag{12}$$

The relative frequency of any elementary cell in fraction i is equal to:

$$\alpha_i = \frac{x_i}{c_i} \tag{13}$$

where x_i is the relative frequency of fraction i.

The grading entropy S is derived by using secondary cells and inserting the relative frequency of the secondary cell α_i:

$$S = -\frac{1}{\ln 2}\sum_{x_i \neq 0} C_i \frac{x_i}{C_i} \ln \frac{x_i}{C_i}, x_i \geq 0 \tag{14}$$

where C_i is the number of the elementary cells in fraction *i,* and x_i is the relative frequency of fraction *i.* The grading entropy *S* is split into the base entropy S_o and the entropy increment ΔS:

$$S = S_0 + \Delta S \tag{15}$$

The base entropy S_0 and the normalized form *A*:

$$S_0 = \sum x_i S_{0i} = \sum x_i i \tag{16}$$

$$A = \frac{S_0 - S_{0\min}}{s0\min_{o\max}} \tag{17}$$

where S_{0k} is the *k*-th fraction entropy (Table 1), which is defined as follows (Table 1):

$$S_{0k} = \frac{\ln c_k}{\ln 2} S_{0k} = k \tag{18}$$

where S_{0i} is the grading entropy of the *i*-th fraction (called here also as abstract diameter *D*), S_{0max} and S_{0min} are the entropies of largest and smallest fractions, respectively. The entropy increment ΔS and the normalized version *B*:

$$\Delta S = -\frac{1}{\ln 2} \sum_{x_i = 0} x_i \ln x_i \tag{19}$$

$$B = \frac{\Delta S}{\ln N} \tag{20}$$

2.3.2. Internal stability

The base entropy S_0 is essentially a mean abstract log diameter which varies between S_{0max} and S_{0min}. The base entropy S_o is a kind of dimensionless mean log diameter, a most probable diameter.

The relative base entropy A (normalised mean abstract log diameter) varies between 0 and 1 and indicates the relative distance of the mean diameter from the maximum-minimum abstract diameter values.

If $A > 2/3$ then enough large grains are present to form a stable soil matrix. If $A < 2/3$ then the coarse particles "float" in the matrix of the fines. The elongated grading curves have a transitional zone.

2.4. The k researches

2.4.1. Nagy [11]

The artificial mixtures of natural soil mixtures were prepared from fluvial soil mixtures for permeability testing. The d_{10} or 10% diameter value of the measurement series were as follows: I. series of measurements d_{10} = 0.004-0.006 mm, II. measuring series d_{10} = 0.006-0.010 mm (III. measuring series d_{10} = 0.010-0.014 mm, IV. measuring series d_{10} = 0.014-0.016 mm.)

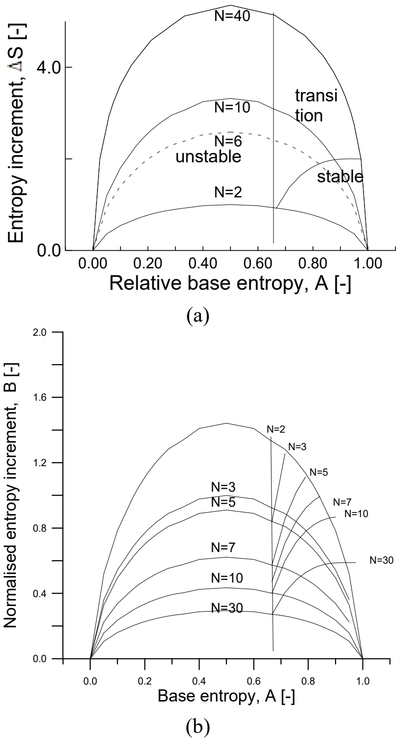


Figure 2. Internal or grain structure stability criterion (a) Partly normalized grading entropy diagram, the lines are the maximum lines (images of the optimal lines) for various N values. (b) The normalized grading entropy diagram for various *N*. The line related to transition zone is dependent on N.

For high-precision measurements, a total of 3 repetitive *k* determinations were performed on 207 samples of falling head test using 74 pre-determined soil mixtures in 4 series of measurements.

The conclusions drawn from the investigations were as follows: the *k* showed a decreasing tendency with the increase of the uniformity index *U*. The range of validity was U < 200-250, suffusion took place if U = 400-500. In this work gradation series 1, 2 were re-examined (grading curve, *k* value).

2.5. Feng et al. [1, 2]

Feng et al [1] studied whether or not the normalised entropy coordinates (*A*, *B*) can be used to predict the coefficient of permeability (*k*). A series of constant head permeability tests performed on 30 laboratory fabricated granular soil samples made from crushed baslat and gritstone. The tested samples had d_{10} ranging from 0.72 to 7.02mm, and the measured $k_{20°C}$ ranged from 3.78 to 501.07 mm/s.

The average gradations examined before and after the test were used to calculate the normalised grading entropy parameter (*A, B*). Considering the gradation ranges of the tested mixtures, the "smallest diameter" d_o were selected as 0.0375 mm in the grading entropy calculation. To remove the potential discontinuity in normalised entropy diagram [9], 'zero' fractions were also introduced in the calculation to better depict the ongoing changes in the gradation within the tested sample mixtures.The plotted normalised grading entropy coordinates of the 30 tested sample mixtures can be categorised into three classes based on their k level (see Fig. 3).

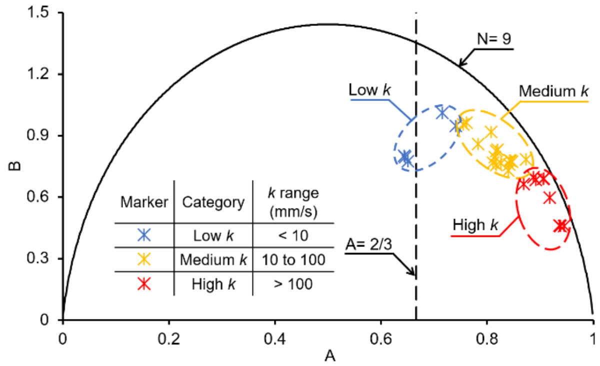


Figure 3. Identified permeability zones shown on the normal-ized entropy diagram (adapted from Feng et al, 2019a) [1]

The multiple linear regression of the measured $k_{20°C}$ with the calcuted A and B values gives:

$$k_{20°C} = 145.47A^{8.9}B^{-2.3}$$
$$R^2 = 0.90,\ n = 30,\ p < 0.0001 \quad (21)$$

In [2] Eq. 21 was modified by introducing the void ratio e:

It is

$$k_{20°C} = 671.83A^{5.59}B^{-1.30}e^{4.59}$$
$$R^2 = 0.96,\ n = 30,\ p < 0.0001 \quad (22)$$

3. Materials and methods

3.1. Materials

Two different families of gradation curves (series 1, 2) consisting of thirty one different samples were fabricated in the original study for falling head testing. For each sample, the PSD was determined by sieving test with the procedure generally followed ASTM and by hydrometer procedure of the Hungarian standard. The results were represented by continuous curves in the traditional way, as shown in Figure 4(a).

To extend the d_{10} range of the grading curves, some additional sets are considered : earlier research gravel permeability tests (set 5, d_{10} = 0.72 to 5,82 mm [1]), a few new, coarse sand mixture data (sets 9 and 10, d_{10} = 0.28 to 1.4, ongoing measurement [17-18], and a set of permeability tests of various authors on transitional soils (set 6 [15]).

In this research the data were reevaluated. The steps were the following. The central moment were used to assess the type of the function using the Pearson diagram. The Weibull function was fitted on measured data and the distribution was extrapolated to the crushing limit. The classical grading entropy parameters and a new spwcific surface parameter were determined.

The four entropy parameters were computed assuming that d_o=6.1E-05 mm. The minimum grain size was limited by a crushing limit around 0.12207 microns (1. 22 E-04 mm), the measured grading curves were extrapolated until this point.

3.2. Methods

3.2.1. Extension of grading curve data

The two different families of gradation curves (series 1, 2) were digitalized fron the original graphs. The central moments of the diameter d were computed, using these, the type of the distribution was determined in the Pearson diagram. A Weibull distribution was fitted to the data (e.g., Guida at al. [14]), the minimum grain size was limited by a crushing limit around 0.12207 microns (1. 22 E-04 mm), the measured grading curves were extrapolated until this point.

3.2.2. Computed parameters

The central moments were computed for the abstract diameters, also. The expected value of the log diameter is the base entropy S_0. The variance of the abstract (or a kind of log) diameter is not used in the grading entropy theory, here it was added as a possible grading entropy parameter and was also examined.

$$Y = \sum x_i S_0^2 - S_0^2 \quad (23)$$

In the reanalysis of the original data, the gradation parameters (D_{10}, D_{50}, D_{60}, D_m, C_u = D_{60}/D_{10}) were determined, moreover, the four entropy parameters were computed (assuming that d_o=6,1E-05 mm) for the data with or without extension, the results were graphically compared.

3.2.3. *Additional soil parameters for permeability*

A kind of equivalent grain diameter is the harmonic mean, dh. It can be defined as the diameter of the sphere which has the same ratio of solid volume /solid surface V_s / Ss as the solid phase as a whole.

In this work some formulae were derived in addition for the specific surface area per mass or volume of a soil which are the total surface area divided by the mass or volume of a given undisturbed sample. These are the mean pore volume and the specific surface area per massor per volume.

The mean pore volume was defined as ρ_v:

$$\rho_v = \frac{V_v}{S_s} = \frac{V - V_s}{6V_s \sum_{i=1}^{N} \frac{x_i}{d_i}} = \frac{e}{6 \sum_{i=1}^{N} \frac{x_i}{d_i}} = \frac{e}{6} d_h \quad (25)$$

where Ss surface area of pores or solid phase (of all fractions), Vv volume of voids in solid phase, e is void ratio= Vv/ Vs, Vs volume of solid phase, the dh is the harmonic mean diameter.

The specific surface area per mass of a soil is the total surface area to the mass of a given volume of grains. The particle surface area is a value in square metre, and the mass is grams, so the units are m^2/g. The specific surface area per mass of a single sphere:

$$S/(V\rho_s) = 6/(\rho_s d) \tag{26}$$

For a volume of soil:

$$S_s / V_s = 6(1-n)/d_h \tag{26}$$

where S_{set} surface area of grains, $V_{s,set}$ volume of solid phase, N fraction , e void ratio, n porosity
The specific surface area per volume is the ratio of the total surface area (m2) to the volume of the soil (m3). The particle surface area per volume unit is so it has units of m-1. For a single sphere:

$$\mathrm{S}/(\mathrm{V}\rho_s) = 6/(\rho_s \mathrm{d}) \tag{27}$$

For a volume of soil:

$$S_{set} /(\rho_s V_{s,set}) = 6/[(1+e)\rho_s d_h] = 6(1-n)/(\rho_s d_h) \tag{28}$$

3.3. Tested correlations

In the reanalysis of the original data, the tradiitional gradation parameters were compared with the statistical parameters. The four entropy parameters computed in 3 different ways (first all extended fractions were taken into account then the presence of possible smaller grains were neglected or substituted by zero value).

The variance of the abstract diameter (self-entropy of the fractions) was not used in the grading entropy theory, here it was also examined.

The regression of of the measured k with various diameter values and specific surface parameters derived here were started to be tested. The multiple linear regressions of the measured k with various grading entropy parameteres, the calculated A and B value, moreover, the calculated base entropy parameter S_0, the entropoy increment ΔS were started to be determined.

4. Results

4.1. Effect of the fines on the entropy parameters

The original grading curves after digitalizationand the Weibull fittings with minimum fraction size 0.00012mm are shown in Figure 4 and in the Appendix..

The extended grading curves for 16 to 18 non-zero fraction sizes were determined either by fitting a Weibull distribution (e.g., Guida at al. [14]) or by assuming smaller zero fractions up to the foregoing actual smallest limit.

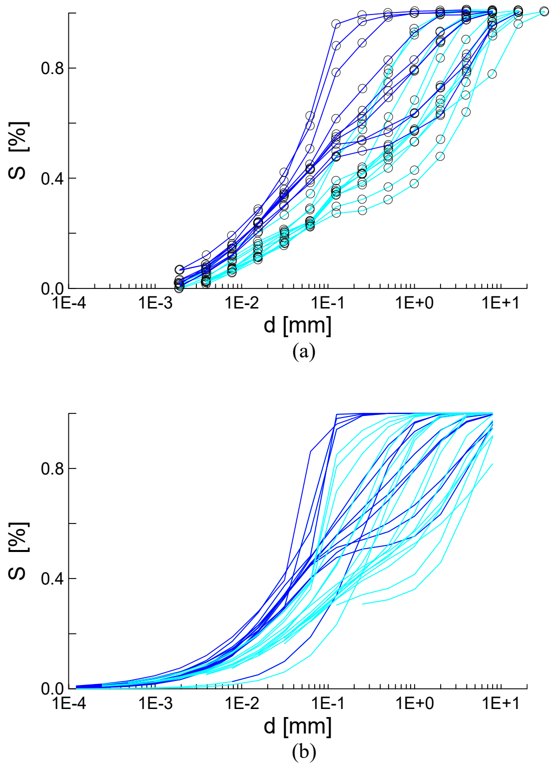


Figure 4. (a) Measured PSDs of gradation types 1 (dark blue) and 2 (light blue) examined. (b) Weibull fitted PSDs of gradation types 1 (dark blue) and 2 (light blue).

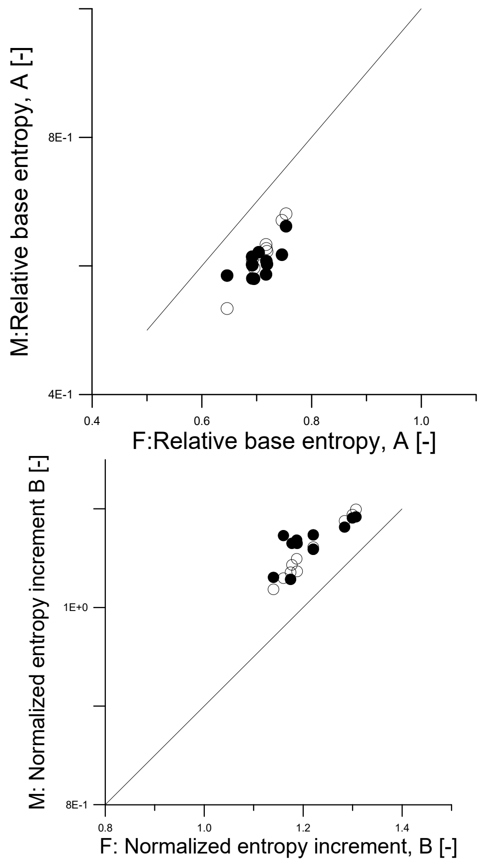


Figure 5. Effect of the small/zero fractions

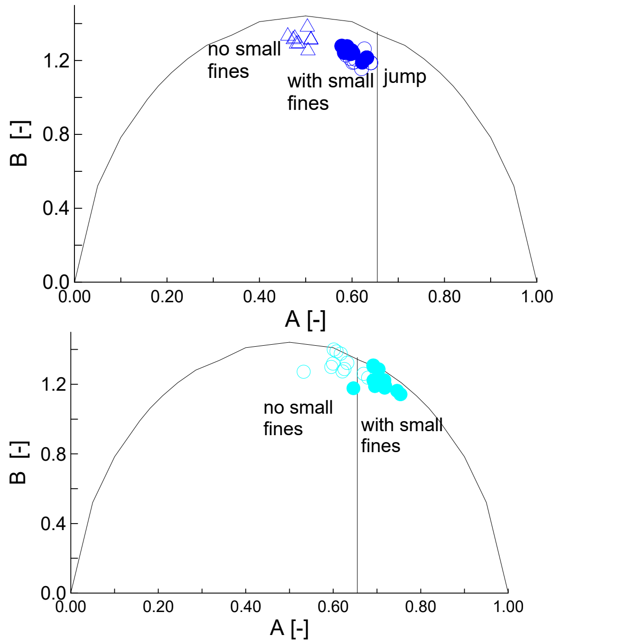


Figure 6. Effect of fines (gradation type 1 (dark blue) and 2 (light blue)) on the normalized and on the non-normalized entropy parameters. The open circles are by assuming smaller zero fractions up to the foregoing actual smallest limit (gave practically the same results as the Weibull data).

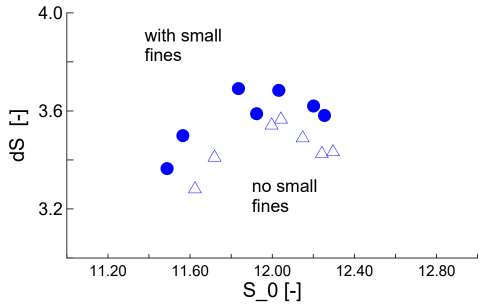


Figure 7. Effect of fines (gradation type 1 (dark blue) nd 2 (light blue)) on the non-normalized entropy parameters (Series 1.2 -1.7, 1.10-1.11).

Computing the normalized grading entropy parameters by taking into account the fines up to N=16 to 18 fractions, according to the results shown in Figures 5 to 6, the gradings fell into the more stable regions. The non-normalized parameters changed significantly if the Weibull fitted curves were considered (the zero fractions did not change their values, see Figure 7). The base entropy So decreased, the entropy incrementcincreased by considering the fines precisely. Therefore, it is essential to assess the precise value of the fines in the grading curve measurement. In the normalized diagram, by assuming smaller zero fractions up to the smallest limit, practically the same results were obtained.

4.2. Comparing grading curve parameters

Subseries (selected curves) 1.2 -1.7, 1.10-1.11 and 2.6,2.7,2.8,2.9,2.12,2.13,2.14,2.15,2.16,2.17,2.18 were used in the analyses.

According to the firstresults shown in Figures 7 to 10, the base entropy S_o showed (a theoretically based) „strong" regression with the parameters d50 and dm. The relative base entropy *A* or the base entropy So showed a series-dependent, strong relationship with *U*. The variance parameter *Y* showed a strong relationship with the entropy increment *ΔS*.

Surprisingly, the relation between parameters d50 and dm was not too strong.

The base entropy S_0 showed a non-unique, series-dependent regression with C_U = *U*. The entropy increment ΔS showed a unique regression with C_U = *U*. The variance parameter *Y* showed a strong relationship with the entropy increment *ΔS* indicating that the entropy increment *ΔS* is basically a variance parameter.

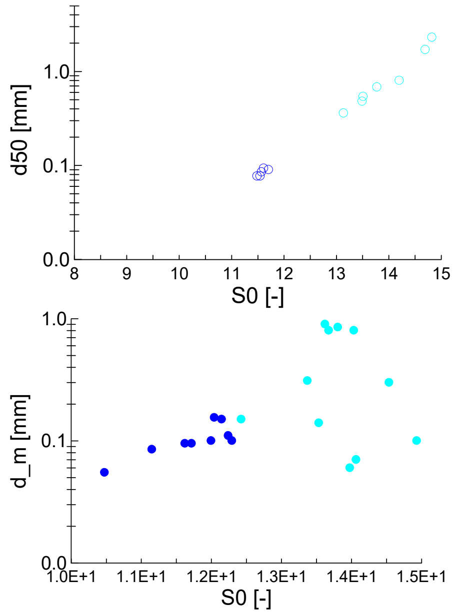


Figure 8. Relations with D_{50}, D_m for PSDs of gradation types 1 (dark blue) and 2 (light blue) examined

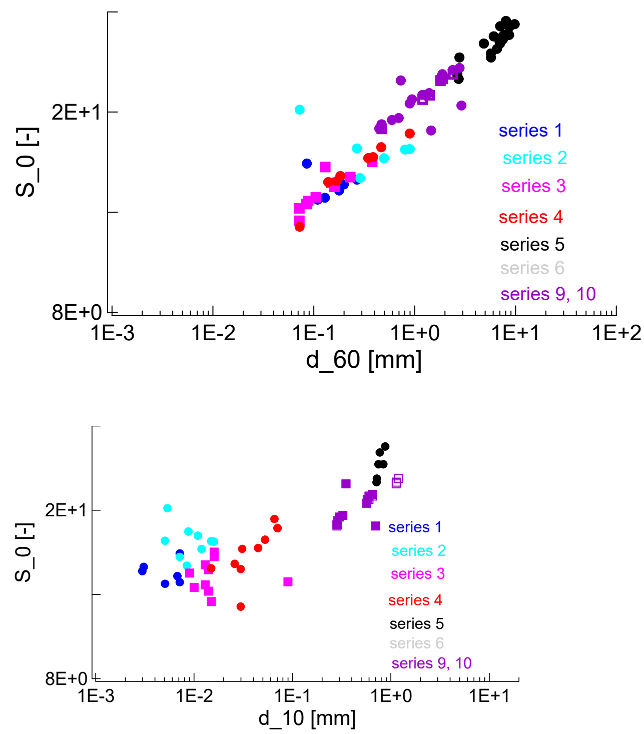


Figure 9. S0 and di (close to mean d it is stronger with larger R2)

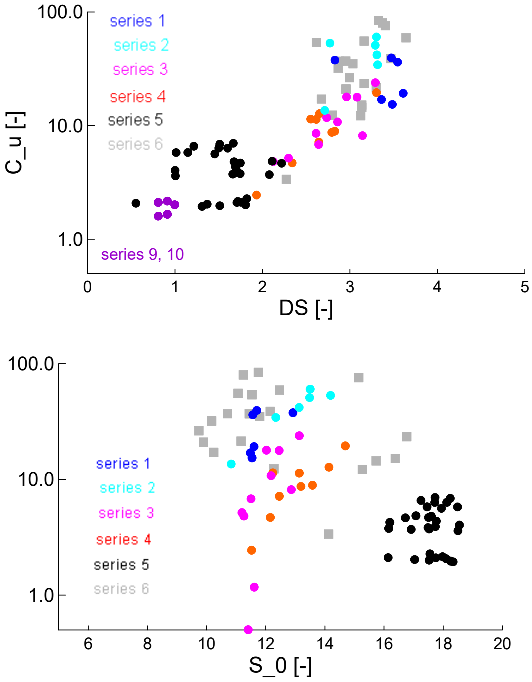


Figure 10. Relations with Cu=U for PSDs of gradation types 1 (dark blue) and 2 (light blue) examined with non-normalised entropy coordinates. The entropy increment ΔS showed a univocal relation. The S_o regression was different for the various series.

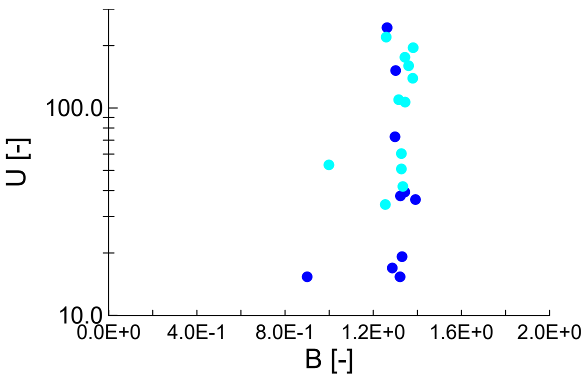


Figure 11. Relations with Cu=U for PSDs of gradation types 1 (dark blue) and 2 (light blue) examined with normalized entropy coordinates.

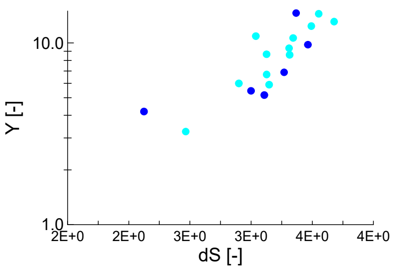


Figure 12. Relations with Y and the entropy increment parameter for PSDs of gradation types 1 (dark blue) and 2 (light blue) examined

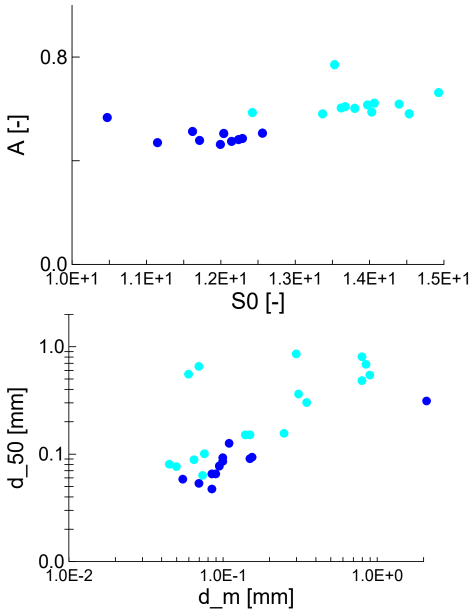


Figure 13. Relations of the two base entropy parameters and for the two empirical mean estimates d50 and dm, for PSDs of gradation types 1 (dark blue) and 2 (light blue) examined

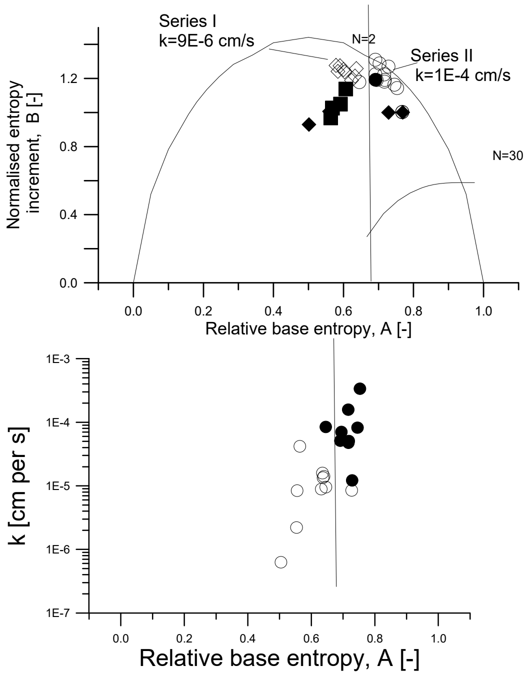


Figure 14. The mean falling head test data on pre-determined soil mixtures in the 2 series of measurements.

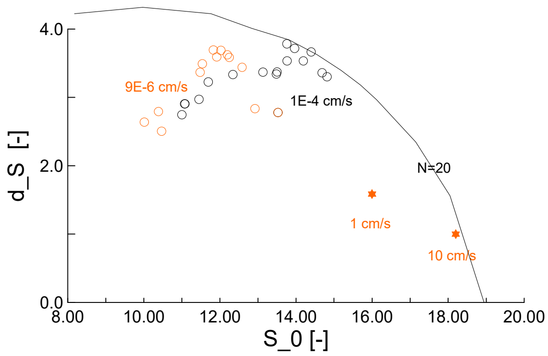


Figure 15. Re-evaluation of data series 1 and 2 of Nagy in the light of literature data of Feng et al [1] in non-normalized diagram.

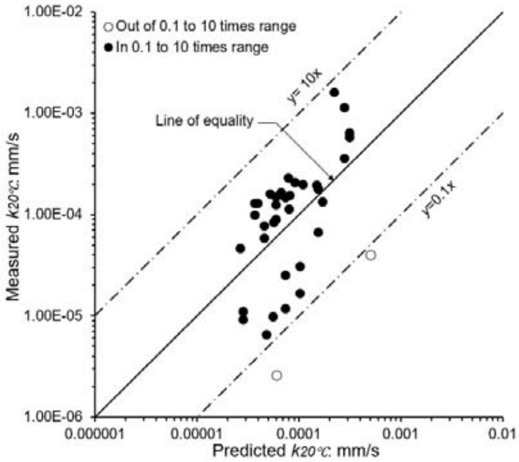


Figure 16. The predicted versus measured k using Eq.25

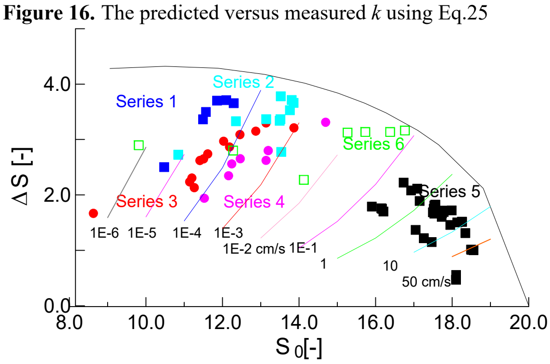


Figure 17. The level lines related to Eq. 35

Table 1. The parameters determined by the Gauss Normal Equations, the variance, coefficient of variation (SD/mean), fitting error 2

	Series 1-2			
	all data		selected data	
		-Bristol		-Bristol
A	-4.497	-4.818	-5.670	-7.330
B	14.210	11.958	10.364	8.332
C	-40.577	-34.719	-28.986	-22.354
VAR(A)	0.150	0.180	0.262	1.964
VAR(B)	1.149	1.016	1.928	7.524
VAR(C)	9.051	7.873	13.585	35.856
Fittingerror -	0.048	0.028	0.019	>0.004

Table 2. The parameters determined by the Gauss Normal Equations, the variance, coefficient of variation (SD/mean), fitting error 2

	Series 1-2
A	-2,71082
B	7,487592
C	-0,22417
D	-26,2253
VAR(A)	2,240154
VAR(B)	1,313213
VAR(C)	0,253004
VAR(D)	12,1185
Fitting error [-]	0,010345

4.3. Multilinear regressions- coefficient of permeability (k)

4.3.1. *Normalised entropy parameters*

The artificial mixtures of natural soil mixtures were prepared from fluvial soil mixtures for the permeability testing. The d_{10} or 10% diameter value of the measurement series used here were as follows: I. series of measurements d_{10} = 0.004-0.006 mm, II. measuring series d_{10} = 0.006-0.010 mm. the mean of the 3 repetitive k determinations related to these soil mixtures in 2 series of measurements.

Using the 2 series of measurements, a strong relationship between A and k moreover between the base entropy parameter S_0 between and k are found (Fig. 12, 13). Unifying the data with some values published by Feng et al 2018, the two data sets showed a consistent dependence on the mean log diameter (first, base entropy parameter S_0) indicating the possibility of the approximate interpolation of a k function in terms of the grading curve (Fig. 14).

Using some elements of data series 1, the previous equation of Feng et al was nicely supported. As follows. Comparing with the results of Feng et al. [1], the two data sets allows the approximate interpolation of a k function in the non normalized diagram.

Eq. 21 and Eq. 22 were examined using data series 1 and 2 from Nagy [10] and gives:

$$k = 0.0012A^{1.10}B^{-8.40} \qquad R^2 = 0.23,\ n = 39,\ p < 0.002 \tag{29}$$

$$k = 0.0065A^{3.12}B^{-3.95}e^{4.62} \qquad R^2 = 0.27,\ n = 39,\ p < 0.001 \tag{30}$$

Eq. 29, 30 generally present the same trend of correlation as Eq. 21, 22 with some degree of variation on the regressed coefficients and exponents (cf. the exponent on the void ratio Eq. 29 and 30). The predicted k is plotted againsted measured k (Fig. 15) shows that 37 out of 39 give predicted k value within 0.1 to 10 times range, 27 out of 39 points give underpredictions of k, while the remainders (12 points) give overpredicted results for k.

4.3.2. *Non-normalised entropy parameters*

The relationship among the base entropy parameter S_0, the entropoy increment ΔS and k is determined in the forms [16]:

$$k = \exp(C_3)\Delta S^{C_1} S_0^{C_2} \tag{31}$$

The predicted versus measured k using Eq.35.
The relationship among the base entropy parameter S_0, the entropoy increment dS, void ratio e and k is determined in the form:

$$k = \exp(C_4)\Delta S^{C_1} S_0{}^{C_2} e^{\,C_3} \tag{34}$$

Series 1 and 2, selected data and, some soils from the University of Bristol research were used for the regression analysis sometimes when indicated ("B data"). Results are shown in Tables 2 and 3.
According to the results, the R^2 was generally greater than 0.6 for selected data.
The parameter C_2 of ΔS is entropy increment is always negative, parameter C_2 related to the S_0 base entropy if generally positive, the fitting is slightly better.

$$k = \exp(\text{-}40.58\ )\ \ \Delta S_0{}^{-4.497} S_0{}^{14.21} \tag{35}$$

$$k = \exp(-26{,}22)\Delta S^{-2{,}71082} S_0^{7{,}487592}\ e^{\ -0{,}22417} \tag{36}$$

4.4. *The diameter variables*

Using these parameters and Feng's data and series 2, the regression equations were determined in Figs. 17 and 18, resp. According to the results, the best R^2 was found for the specific surface variable. The R^2 was larger for gravel and in this case if the entropy parameters were used.

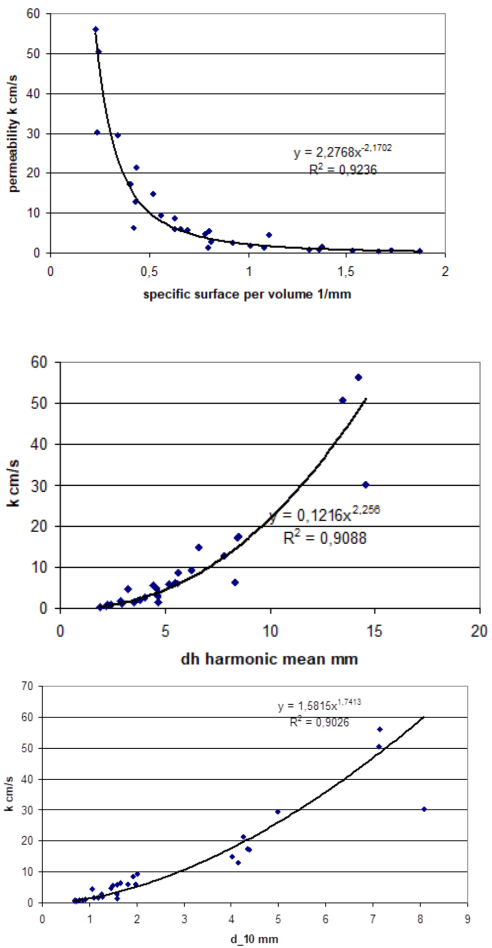


Figure 18. Re-evaluation of data of Feng in terms of various diameter values (specific surface parameter (containing density info), harmonic mean d_h and d_{10} with R2= 0.92, 0,91 and 0.90 resp.).

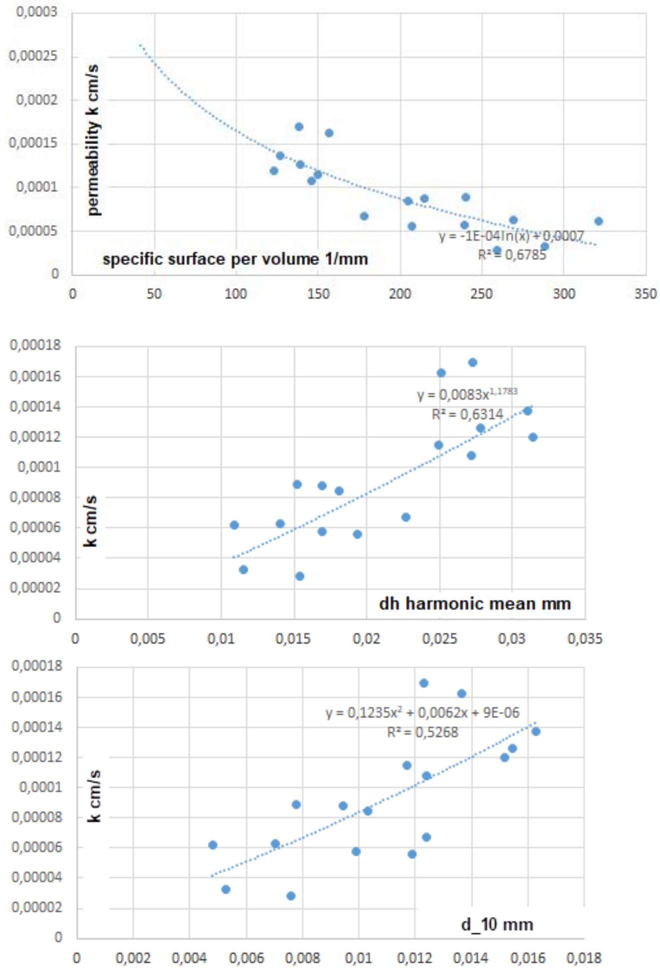


Figure 19. Re-evaluation of data series 2 of Nagy in terms of various diameter values specific surface parameter (containing density info), harmonic mean d_h and d_{10} with R2= 0.68, 0,63 and 0.52 resp.).

5. Discussion

The aim of the research is to introduce a statistically consistent, numerically more effective parameters which can be used both for a single empirical grading curve and for the space of the all possible empirical grading curves which is needed if any interpolation is made in terms of the grading curves. The very first results are summarized here.

5.1. Traditional grading curve parameters

5.1.1. Generally used parameters

The grading curve is a statistical distribution of log diameter d with respect to dry weight. The empirical distribution function can be determined by the sieving of the granular matter with large enough grains, it is made with sieves that have "the classical sieve hole diameters", which are resulted by successive multiplication or division with a factor of 2, starting from eg. 1 mm.

On the basis of the sieving test, a graphically interpolated, continuous function is given in terms of log d to determine graphically some statistical "quantile" diameter values (d_{10} and d_{60} and d_{50}) and at the unique inflexion point – if exists – a "dominant" diameter value (d_m). These statistics are given then in arithmetic d scale instead of the log scale.

The coefficient of uniformity U or C_u - as ratio of d_{60} and d_{10} - is then computed which is similar to the standard deviation: if it is small then the d values are located around the d_{50} or d_m which approximate the mean d.

5.1.2. Specific surface parameter

More precise parameters are the harmonic mean and specific surface since these are based all measured data.

In this work some formulae were derived in addition for the specific surface area per mass or volume of a soil which are the total surface area divided by the mass or volume of a given undisturbed sample.

These were compared with the using series 2 and Feng's gravel series on the basis of permeability data with d_{10}. According to the results, the value of R2 was larger for the gravel series than for the fine sand-silt series. Within each series, the best was the sepcific surface area and the worst was the d_{10}.

5.2. Grading entropy

In the grading entropy theory the measured grading curve is considered as a finite, discrete distribution function with a non-uniform cell system in the arithmetic d scale and uniform in the log scale. The primary cell system is defined as an abstract fraction system using the log value of the classical sieve hole diameters. A secondary cell system is defined with equal increments on the arithmetic d scale, using this, a uniform discrete distribution is assumed within each fraction. Using the concept of the entropy of a discrete distribution, a specific entropy term was derived which splits into two parts, called entropy parameters.

5.2.1. Grading entropy parameters

The main observations of the ongoing research are summarized as follows. The base entropy is similar to a mean log d value. The entropy increment may characterize the spreading of the data around the mean. It has a physical meaning, also since it expresses the contact between the spontaneous processes and the entropy principle. (It was proved experimentally that there is a relations for the natural processes with the entropy principle [11]).

The relative base entropy is useful since it may reflect the amount of the large grains. In case the amount of large grains is great enough then they may form stale structure, so the normalised parameter may reflect the microstructure and internal stability.

The normalised he entropy increment is useful since the range difference originated from different fraction number N approximately disappears.

The entropy increments ΔS and its normalised version B, reflect the actual effective number of fraction within the mixture – as a more precise coefficient of uniformity C_u. Its maximum is ln$N$/ln2 and 1/ln2, respectively.

The entropy increment ΔS expresses the 'disorder' due to the fractons an effective fraction number, its maximum is equal to ln$N$/ln2. The B varies between 0 and 1/ln2. It is zero for a single fraction and is maximal where each relative frequency is the same (uniform distribution).

5.2.2. Interpolation

To interpolate a function over the space of the grading curves, it is important to characterize the space of the grading curve. The B, ΔS; and A, S_0 entropy coordinates can be transformed into each-other, resp.; so the equations formulated in terms of normalised coordinates can be transformed into non-normalised coordinates. Eq. 24 , 25 can theoretically be transformed into a form using ΔS; and S_0 instead of B, and A.

For a given relative base entropy parameter A [-], the maximal normalized entropy increment is related to the possibly most uniform distribution which is a fractal distribution for each A value with fractal dimension varying between minus to plus infinity [14].

In the research, 15 laboratory tests on saturated permeability were conducted on fractally distributed sand mixtures (which are mean grading curves for a given A value and $N=2$) with given composition. In the future part of the research, larger N valued fractal gradings will be further tested.

6. Summary

6.1. The effect of fines

To asses the effect of fines the four entropy parameters computed in 3 ways (leaving out the end of the grading curve, using zero relative frequency for the fines and taking into account the fines) were compared. According to the results, the non-normalized parameters changed significantly if the fines were considered. The base entropy S_o decreased, the entropy increment ΔS increased by considering the fines precisely.

The normalized parameters fell into more stable regions by taking into account the fines up to N=16 to 18 fractions while otherwise they fell into the unstable region (either by model fitting or assuming zero fractions). It follows that it is essential to assess the precise value of the fines in the grading curve measurement.

6.2. The relations of the entropy coordinates and the traditional parameters

The base entropy S_0 showed (a theoretically based) strong, unique relationship with the parameters d_{50} and weaker with dm. The base entropy parameter S_0 showed for each series a unique, strong, monotonic increasing relationship with U, therefore, the original conclusions can be reformulated for each series for S_0.

The grading entropy type variance parameter Y showed a strong, unique relationship with the entropy increment ΔS possibly due to the shift symmetry of the latter. The entropy increment ΔS with shift symmetry is basically a variance type parameter, being the largest for the uniform distribution.

6.3. The regressions

The k values determined by the two research projects plotted in the non-normalised entropy diagrams consistenly showed an increasing tendency with increasing mean grain size (and mean pore size). Results allow the approximate interpolation of a k function in the non normalized diagram. Further research is suggested on this.

The entropy coordinates are pseudo-metrics.The B, ΔS; and the A, S_0 entropy coordinates can be transformed into each-other, resp.; so the equations expressing the saturated k value in terms of normalised coordinates can be transformed into non-normalised coordinates which will be done in a further research.

Using the grading curve parameters (d10, specific surface, harmonic mean diameter) and Feng's data, the best-fit non-linear regression equations were determined. According to the results, R^2 was larger if the entropy parameters were used. Concerning series 1 and 2, the regression was weaker, the R^2 was less than 0.6 if all data were used. Further research is suggeted on this difference related to clays (seris 1 to 2) and gravel (Feng's data) in the regression analyses.

7. Conclusions

The geotechnical correlations are generally formulated in terms of diameter values such as d_{10}, d_{30}, d_{50} and d_{60} and or their ratios. These are related to one single point of the grain size distribution. More precise parameters are the harmonic mean and specific surface in terms of correlations with the permeability.

The four classical grading entropy parameters are based all measured data, therefore, these are more precise than the single diameter values. In addition, the four grading entropy parameters have several additional mathematical and physical meaning which can be used in regression forms. Only two are mentioned here.

1. The relative base entropy A is the number to characterize the internal structure of the grains, which can be stable/unstable if water is flowing through. In other words, this parameter well classifies the soils with respect to internal structure.
2. The grading curves and the regression equations can be represented in the entropy diagram with points and level lines, respectively. The permeability zones of [1] are in accordance with the level lines presented here.
3. The conclusions drawn from the original investigations were as follows: the k showed a decreasing tendency with the increase of C_u and increasing tendency with d_{10}. The original conclusions were reformulated in terms of S_0 and and ΔS and strong regression was found between ΔS and the C_u, and between S_0 and d_{10}.

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