# An investigation of the role of leadership in consensus decision-making 

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#### Abstract

Leadership is a widespread phenomena in social organisms and it is recognised to facilitate coordination between individuals. If the role of leadership in group foraging or swarm movement is well understood, it is not clear if leaders would also benefit more complex forms of coordination. In particular, a number of organisms coordinate by consensus decision-making, where individuals explicitly communicate their opinions until they converge toward a common decision. Taking inspiration from physical sciences, we extend a consensus formation model to integrate leaders, which we define by three traits: persuasiveness, talkativeness, and stubbornness. We use numerical simulations to investigate the effect of the number of leaders and their characteristics on the time a group spends to reach consensus, and the bias in final decision. We show that having a minority of influential individuals (leaders) and a majority of influenceable individuals (followers) reduces the time to reach consensus but biases the decision towards the preferences of the leaders. This effect emerges solely from the differences in individuals' personality traits, with the most determinant trait being the talkativeness of the individuals.


Overall, we provide a comprehensive investigation of the effects of leaders and their traits on consensus decision-making.

Keywords: leadership, consensus decision-making, opinion dynamics, coordination

## 1 Introduction

Leadership describes a phenomena exhibited in many social organisms, where few individuals - leaders - modify the behaviours of other individuals followers (Smith et al., 2016). Examples of leadership in nature go from group movements guided by a few individuals (Couzin et al., 2005), to the complex hierarchical structures exhibited by human societies (Diefenbach and Sillince, 2011; Day, 2013). A major goal of life sciences research on leadership is to describe and understand the effect of leadership on the functioning of the group and the success of its members. Understanding how leadership traits affect the success of both leaders and followers is particularly important to understand why leadership has emerged. Yet, some roles of leaders are still hard to understand. In particular, leaders are recognised to facilitate group coordination, but it is not clear if and how leaders would do so when groups coordinate by consensus decision-making.

Everyday, social groups have to take collective decisions to coordinate their actions and activities. Examples encompass initiation of group departure in swans (Black, 1988), choice of nest location in bees (Seeley and Buhrman, 1999) or collective hunting in humans groups (Alvard and Gillespie, 2004). Some social animals achieved coordination using relatively simple interactions patterns, from which the role of leader results. For instance, coordinated swarm behaviour is the result of a majority of individuals following their neighbours and an individual in the front leading the group.

However, in some organisms, coordination may not be accomplished by interaction rules alone but rather by an active process of consensus decision-making,
in which individuals communicate their opinions until they converge toward a common opinion. This form of coordination can be observed on a day-to-day basis in human groups, whether it is in the parliaments of complex states or in the meetings of hunter-gatherers tribes (Von Rueden et al., 2014; Boehm, 2001). Moreover, there is a number of non-human organisms exhibiting consensus decision-making (Conradt and Roper, 2005) using ritualized movement (Seeley and Buhrman, 1999), vocalizing (Stewart and Harcourt, 1994), or even sneezing (Walker et al., 2017). Yet, the lack of a mechanistic description of consensus decision-making has limited the investigation of the role of leaders in this process.

Investigation the process of consensus decision-making has often been limited because of its complexity. However, consensus decision-making is a well-known process in physical science, where it has been modelled by opinion formation models. Opinion formation models describe the sequence of communication during which individuals transmit their opinions, and provide a stylised representation of the spread of opinions in a population (Castellano et al., 2009).

Famous opinion formation models include the Degroot model (Degroot, 1974) and the voter model (Clifford and Sudbury, 1973; Holley and Liggett, 1975) but they now encompass a large set of models (Castellano et al., 2009) which have been successfully applied in diverse fields, for instance to understand the adoption of innovation (Valente, 1996), the spread of extremism (Deffuant et al., 2002), or the polarisation of opinions (Rocca et al., 2014) (see (Dong et al., 2018) for a review specific on consensus processes in opinion formation models). Nevertheless, their applications to the topic of leadership in life sciences has been so far limited.

Previous theoretical work have shown that heterogeneity in individuals' personality traits could strongly affect the time a group spends to reach consensus (Mobilia et al., 2007; Galam and Jacobs, 2007; Jalili, 2013; Gavrilets et al., 2016). This could explain the benefit that leaders provide to coordination because the time to reach consensus can be costly, either because time itself carries a cost, e.g. resources get depleted, or because time constraints will force indi-
viduals to take a sub-optimal decision (Chittka et al., 2009; Franks et al., 2003) e.g. a quick decision has to be taken during an intergroup conflict.

Nonetheless, it is hard to draw general conclusions on the effect of leaders on consensus decision-making based on previous work. For instance, the presence of stubborn individuals could either slow down the consensus (Mobilia et al., 2007; Galam and Jacobs, 2007), or speed up the consensus (Pérez-Llanos et al., 2018). Persuasive individuals could allow consensus to be reached quicker, but only if the persuasive individuals can also signal to a high number of individuals (Jalili, 2013; Gavrilets et al., 2016).

The lack of general conclusions from these models is explained by these models focusing on different questions, such as the role of one single perturbing individual like a zealot (Mobilia et al., 2007), or on the effect of diversity of traits on the consensus seeking process (Gavrilets et al., 2016). Thus, there is still the need for a comprehensive investigation of the effect of leaders on consensus decision-making. To fill this gap, this paper aims to (i) clearly demonstrate and quantify the benefit and cost of leaders on consensus decision-making, and (ii) identify under which conditions, that is number of leaders and characteristics of leaders, leadership would provide these benefits and costs. To do so, the analysis and model presented here differs in three points from previous work. First, we consider the three key characteristics previously identified in models and observed in leaders profiles (Judge et al., 2002): persuasiveness ${ }^{1}$, stubbornness and talkativeness, while previous work often focus on a single trait. Second, we divide the group between leaders and followers and consider all possible compositions of groups, rather than the presence of a single leader. This allow us to investigate how multiple leaders interact. Third, we vary independently the traits of leaders to better understand which traits, i.e. persuasiveness, talkativeness and stubbornness, underlie the effects of leadership on consensus

[^0]decision-making. This allows to clarify previous results that could appear to draw contradictory conclusions.

We investigate these questions in an opinion formation model developed by Gavrilets et al. (2016), in which we can vary the number of leaders and their characteristics. We do not integrate the knowledge of individuals or consider that one potential decision is more efficient than another because we want to clearly identify if influential leaders provide a benefit to coordination tasks, where there are multiple choices providing optimal but equal payoffs (Schelling, 1960). We also want to clarify if leaders can provide an intrinsic benefit to the consensus decision-making besides their knowledge or skills. Doing so, we follow the definition of leaders as individuals occupying a special position in the decision-making hierarchy and who have disproportionate influence over group goals and decisions, rather than individuals being more competent individuals (Von Rueden et al., 2014; Van Vugt et al., 2011; Garfield et al., 2019a). The role of knowledge in leadership has already been well explored (Gavrilets et al., 2016) and there is evidence that human leaders provide a benefit besides their knowledge (Calvert, 1992; Van Vugt et al., 2011), as shown by post Neolithic leaders in human societies taking decisions on a wide range of topics.

## 2 Model definition

We use an opinion formation model developed in previous work (Gavrilets et al., 2016). It is a model which consists of a sequence of discussions between individuals until their opinions are close enough, i.e. the group has reached a consensus. Individuals are described by an opinion $x$. We consider that there is a spectrum of opinion and thus, $x$ is a continuous value defined between $[0,1]$.

The opinion $x$ describes a generic opinion of an individual on how to realise a collective task, e.g. hunting party direction, time for group departure or value of a law. In addition to the opinion $x$, individuals are also described by three continuous traits: (i) persuasiveness $\beta$, i.e. the capacity of one individual to modify the opinion of another individual towards its own opinion, (ii) stubbornness $\gamma$,
i.e. the reluctance of an individual to change its opinion, and (iii) talkativeness $\theta$, i.e. the propensity that an individual communicate with another individual whether it is by talking, vocalising or doing ritualised movement. A large part of our analysis is looking at cases where these traits vary together. Empirical evidence demonstrates that these three traits are correlated in leaders personalities (Judge et al., 2002), and they have been identified in previous models as key factors in explaining how leaders affect consensus time (Gavrilets et al., 2016; Jalili, 2013). Thus, we also define the trait $\alpha$ which is the value of these three traits when they are equal $\beta=\gamma=\theta=\alpha$. $\alpha$ is defined as the individual capacity to influence the collective decision (in short influence). To study the effect of social organisation on collective decision-making, we divide individuals into two profiles: leader L, and follower F.

We consider a population of $N$ individuals. At the beginning of the opinion formation model, the values of opinion $x$ are randomly generated. Each time step is defined by one discussion event during which one individual, i.e. the speaker, communicates to another individual, i.e. the listener. The probability $P$ of an individual $i$ to be chosen as a speaker is an increasing function of its talkativeness $\theta$ as follows:

$$
\begin{equation*}
P_{i}=\frac{\left(\theta_{i}\right)^{k}}{\sum_{n=1}^{N}\left(\theta_{n}\right)^{k}} \tag{1}
\end{equation*}
$$

The parameter $k$ scales how much the probability to talk depends on the talkativeness of an individual. A high value means that the probability to talk depends mostly on talkativeness, while a low value means that other parameters are more important in determining the speaker, e.g. there are rules to enforce equal access to speech as in small-scale societies or contemporary inclusive meetings. In this paper, we use a high $k=4$ as we want to study the effect of talkativeness in the absence of other factors.

We assume that every individual can be chosen as a listener, i.e. the social network is a complete network, because we are interested in short time-scale decision-making rather than the long time-scale spread of opinions. We also consider that individuals interactions are not limited to individuals with close


Figure 1: Probability for a single leader in a group with 999 followers to be chosen as a speaker as a function of its talkativeness $\theta_{L}$ and the scaling parameter $k$.
opinions (as in models with bounded confidence (Deffuant et al., 2002)) because this model describes a consensus-seeking process where individuals are willing to convince each other. During a communication event, a listener v updates its preference to a value $x_{\mathrm{v}}^{\prime}$ following the equation below, where v represents the listener and $u$ the speaker:

$$
\begin{equation*}
x_{\mathrm{v}}^{\prime}=x_{\mathrm{v}}+r\left(\frac{\beta_{\mathrm{u}}}{\gamma_{\mathrm{v}}}\right)\left(x_{\mathrm{u}}-x_{\mathrm{v}}\right) . \tag{2}
\end{equation*}
$$

The parameter $r$ represents the base update rate, i.e. how much a listener will update its opinion if the speaker has the same characteristics than itself. We use a ratio relationship between persuasiveness, $\beta$, and stubbornness, $\gamma$, as in previous work (Gavrilets et al., 2016) because it guarantees the following condition: The change of opinion resulting from a leader communicating to a follower is higher than followers communicating to followers (or leaders to leaders), which is higher than a follower communicating to a leader. The traits $\beta$ and $\gamma$ are defined on $\left[1, \frac{1}{r}\right]$ so that an individual with the highest persuasiveness $\beta$ talking to an individual with the lowest stubbornness $\gamma$ convinces the individual in one event. The talkativeness $\theta$ is also defined on $\left[1, \frac{1}{r}\right]$ so it can be varied

Opinion formation models are commonly studied using analytical methods, by which are calculated exact solutions to the system. However, these approaches are difficult in presence of heterogeneity in the population, which is the case here as individuals have different values of influence. Thus, we implement the model as an individual-based model and use numerical simulations to analyse it. There are two features of the consensus decision-making that leaders could affect and that we measure in the simulations. First, leaders could affect the time the group spend to reach consensus, which is described by $t^{*}$. Second, leaders could also bias the final decision. To measure this bias, we consider that the initial opinion of individuals reflect their preferences, and we measure how close the final decision is from the preferences of all individuals. We then look at the distribution of this distance across individuals. More formally, we define the realised influence $\alpha_{r}$ of an individual $i$ in a simulation run $j$ :

$$
\begin{equation*}
\alpha_{r(i j)}=1-\left|x_{i j}(t=0)-x_{j}^{*}\right| \tag{3}
\end{equation*}
$$

The realised influence of an individual $\alpha_{r}(i)$ is the average of the realised influence of this individual $i$ across 500 consensus decision-making events. Unlike the influence $\alpha$, realised influence $\alpha_{r}(i)$ depends of the influence of other individuals in the group. For instance, a leader would have a high realised influence in a group of followers, but low realised influence in a group with many other leaders. We measure the bias in final decision by the Pearson's moment coefficient of skewness (in short skewness) of the distribution of the realised influence across individuals. A high skewness represents a biased decision, i.e. the decision is close to the preferences of a minority of individuals and far from the preferences
of a majority of individuals. A skewness of 0 represents a fair decision, i.e. the decision is equally close to the preferences of all individuals.

We focus on the effect of the following parameters: (i) the number of leaders and (ii) the influence of leaders $\alpha_{\mathrm{L}}$. In addition, we study the effect of the consensus threshold $h$ because this parameter controls how global is the consensus. Finally, we vary the three traits independently in a group with one leader to better understand how each trait contributes to the effects of leader on the consensus decision-making. The influence of followers $\alpha_{F}$ is set to the minimum value 1 and the influence of leader $\alpha_{\mathrm{L}}$ can vary between $\left[1, \frac{1}{r}\right]$. When other parameters than the influence of leaders is varied, the default influence of leaders is $\alpha_{\mathrm{L}}=5$. The other default parameters are for the consensus threshold $h=0.05$, the base update rate $r=0.1$ and group size $N=100$. The results presented are the mean across 500 replicates for each set of parameters presented. The error bars or ribbon represent the standard deviation from the mean rather than the standard error from the mean because the variance between different runs is important.

## 3 Results

Figure 2.A shows the main result: the presence of a minority of influential individuals and a majority of influenceable individuals reduces the time a group spend to reach consensus. Importantly, the differential quality of information that leaders might posses, and which might lead to a group with hierarchy making better decisions, is not required to get this result. Figure 2.A shows that the shortest time to consensus is obtained in presence of a single leader and that the time to consensus is reduced much less in the presence of multiple leaders. In fact, in some cases groups with multiple leaders can spend more time to reach consensus than a group of individuals of equal influence .

The relationship between time to consensus and numbers of leaders can be derived from the formula in (Gavrilets et al., 2016), for the special case when leaders have an extremely high probability of talking.


Figure 2: A. Time to consensus as a function of numbers of leaders. The influence of leaders is equal to $\alpha_{\mathrm{L}}=10$. B. Density distribution of individual opinion as a function of number of discussion events for different number of leaders: from top to bottom $0,1,2,5$. For illustration, the difference between the opinions of leaders are set to be maximum and equidistant. The plot represents results for a single run. The parameters used are $\alpha_{\mathrm{F}}=1, r=0.1, h=0.05, N=$ 100. Black area represents opinions that no individuals currently hold.

$$
\begin{equation*}
t^{*} \sim \frac{2 N}{r}\left(1-\frac{1}{N_{\mathrm{L}}}\right), \text { for } l \geq 1 \tag{4}
\end{equation*}
$$

In the absence of leaders, the time to consensus is $t^{*} \approx \frac{2 N}{r}$. We see that adding a single leader strongly speeds up consensus, but this benefit is quickly reduced when more and more leaders are added to the group.

Figure 2.B gives an illustration of the dynamics of the model. It shows that in the absence of leaders, or with a single leader, individuals' opinions consistently converge. This homogeneous convergence pattern results in low variance in the time to consensus across the different runs as shown in Figure 2.A. These results suggest that the long time to consensus in groups of individuals of equal influence is mainly due to a slow convergence. The presence of a single leader
speeds up this process as the leader quickly convinces the majority of the group.
Figure 2.B shows that the presence of multiple leaders creates a more heterogeneous pattern of convergence. The presence of two leaders results into two clusters of opinions with the majority of followers switching from one leader to another: leaders alternatively convince individuals from the group but neither leader has enough followers to reach consensus. When more than two leaders are present, the majority of opinions fluctuates between the different leaders. This heterogeneous pattern of convergence results in high variance in the time to consensus between runs as shown in Figure 2.A. This result shows that the time to consensus in groups with multiple leaders is highly dependant of the leaders' initial opinions. When leaders have similar opinions, they quickly convince the rest of the group and it results in a short time to consensus. When leaders have diverse opinions, it results in a slow consensus. This effect is illustrated in simulations shown in Supplementary Figure 1, where the opinions of leaders are set to be the most different from each others. In this case, the time to consensus with multiple leaders is in average worse than the time to consensus in the absence of leaders. This is because multiple leaders (i) are slower to be convinced, (ii) increase divergence by convincing followers towards extreme opinions and (iii) convince followers from other leaders. Unlike groups of equals, longer time to consensus in groups with multiple leaders is due to conflict between leaders rather than a slow convergence.

The previous result considered only the most extreme form of leaders with leaders having the highest influence $\alpha_{\mathrm{L}}=10$. We now investigate different values of leaders influence. The Figure 3 shows that the main result is consistent across different values of leaders influence $\alpha_{\mathrm{L}}$ : The presence of a minority of influential individuals and a majority of influenceable individuals reduces the time a group spend to reach consensus. Figure 3 shows that when leaders are less influential, the shortest time to consensus is obtained in presence of multiple leaders, unlike previous results with highly influential leaders in which a single leader has the shortest time to consensus. The detrimental effect of multiple leaders is not observed when leaders have low influence because leaders convince each other


Figure 3: Time to consensus as a function of numbers of leaders and the influence of leaders $\alpha_{\mathrm{L}}$. The time to consensus for a group with single leader is highlighted in red. The parameters used are $\alpha_{\mathrm{F}}=1, r=0.1, h=0.05, N=100$.
relatively quickly. Once their opinions are close, they act as a single strong leader which quickly convinces the rest of the group. Groups with a single leader with low influence spend more time to reach consensus simply because the leader is less efficient at bringing the opinions of others toward its own. However, across different values of leaders influence, the shortest time to consensus is obtained in presence of one single extremely influential leader.

The above results focus on the time to consensus and demonstrate the beneficial side of leaders which reduce the time that a group spend to reach consensus. However, the final decision resulting from the consensus is also important and could be affected by the presence of influential individuals. To investigate this effect, Figure 4 presents the skewness of the distribution of realised influence, i.e. how close is the final decision from the initial opinion of an individual. A higher skewness represents a strong bias of the decision toward a minority of individuals. Figure 4 shows that leaders bias the decision: a minority of influential


Figure 4: Skewness of the distribution of realised influence $\alpha_{r}$ as a function of the number of leaders and the influence of leaders $\alpha_{\mathrm{L}}$. The skewness for a group with single leader is highlighted in red. The parameters used are $\alpha_{\mathrm{F}}=1, r=$ $0.1, h=0.05, N=100$.
individuals and a majority of influenceable individuals leads to a high skewness of the distribution of realised influence. This result is consistent across different values of leaders influence except when leaders have very limited influence $\alpha_{\mathrm{L}}=1.5$. The highest bias is obtained for groups with one single leader. This is because influential individuals efficiently propagate their opinions (thanks to their high persuasiveness and talkativeness), and maintain their initial opinions longer than followers (thanks to their stubborness). Ultimately, the leaders are able to pull the final decision toward their own preferences. In conclusion, the results show that there is a trade-off between time to consensus and fairness of the decision, i.e. how representative the decision is of the opinions of all group members.

We consider here that only global consensus is possible, i.e. the whole group agrees on the decision. Nonetheless, we can vary the consensus threshold $h$ to
allow for a more or less strict consensus, i.e. divergent opinions are more or less accepted. Supplementary Figure 2 shows that a higher consensus threshold reduces the time to consensus, in particular in absence of leaders or in presence of multiple leaders. Yet, the main results are consistent across different value of consensus threshold $h$ : the presence of a minority of influential leaders results in shorter time to consensus but biased decision. Consensus threshold has a limited effect on the skewness of the distribution of the realised influence. This is because a higher consensus threshold leads to an early end of the consensus process, time at which the decision is already biased. Indeed, influential individuals quickly bring the opinions of others toward their own and the late stage of the consensus process consists of the leader convincing the last remaining individual.

We now vary the traits independently to understand how each trait contribute to the effects of leaders on the consensus decision-making. Figure 5 shows that the time to consensus is highly reduced when the leader is both persuasive (high $\beta_{L}$ ) and talkative (high $\theta_{L}$ ) (first row). In other words, talkativeness and persuasiveness interaction is the main factor reducing time to consensus. For instance, when talkativeness is high (right column), an increase in persuasiveness results in a strong decrease in the time to consensus. When talkativeness of leaders is equal to followers (left column), an increase in the persuasiveness of the leader does not appear to affect the time to consensus. This result shows that talkativeness $\theta_{L}$ is a crucial trait and that the effect of persuasiveness of leader $\beta_{L}$ depends of the talkativeness. This is because talkativeness sets how much a leader communicate and thus, how much a leader exerts its persuasiveness on others.

An intuition behind this result can be obtained using the formula for time to consensus considering that individuals are equal in talkativeness, shown in (Gavrilets et al., 2016). This formula states that the time to consensus is proportional to $\frac{1}{\beta H(1 / \alpha)}$ with $H$ defined as the harmonic mean. If we consider that persuasiveness and stubbornness are equal, that is $\beta=\alpha$, this formula reduces to 1 and the time to consensus becomes independent of the persuasiveness and


Figure 5: Time to consensus as a function of leader persuasiveness $\beta_{\mathrm{L}}$, talkativeness $\theta_{\mathrm{L}}$ and stubbornness $\gamma_{\mathrm{L}}$ in a group with a single leader. The parameters used are $\alpha_{\mathrm{F}}=1, r=0.1, h=0.05, N=100$.
stubbornness of individuals. In other words, the benefit for adding a persuasive individual is exactly cancelled by the addition of a stubborn individual. Talkativeness tilts the balance by increasing the number of times an individual talks (which amplifies the benefits of persuasiveness) compared to the number of times an individual is talked to (which decreases the cost of stubbornness).

Figure 5 shows that modifying the stubbornness $\gamma_{L}$ of the leader has a limited effect on the time to consensus, especially when leaders are already very talkative. We find similar results in Supplementary Figure 3 which shows that adding leaders who are talkative, persuasive but easy to persuade, still reduces the time to consensus. This is because when the leader is talkative,
the consensus decision-making consists mostly of the leader convincing others rather than individuals convincing the leader. Nonetheless, the stubborness $\gamma_{L}$ increases the variance between runs when the talkativeness of the leader is low and when persuasiveness of the leader is high (bottom left panel). This is because a stubborn and persuasive leader is (i) longer to be convinced but (ii) can also bring back other individuals to its opinion, which is far from the emerging consensus.


Figure 6: Skewness of the distribution of realised influence as a function of leader persuasiveness $\beta_{\mathrm{L}}$, talkativeness $\theta_{\mathrm{L}}$ and stubborness $\gamma_{\mathrm{L}}$ in a group with a single leader. The parameters used are $\alpha_{\mathrm{F}}=1, r=0.1, h=0.05, N=100$.

Figure 6 shows how the three traits of the leader bias the final decision. The results show that the level of talkativeness of the leader $\theta_{L}$ strongly affects the bias in decision. For instance, groups with very talkative leader (right
column) has a very skewed distribution of realised influence independently of the persuasiveness or stubborness of the leader. As previously, persuasiveness $\beta_{L}$ and talkativeness $\theta_{L}$ interact. For instance, when talkativeness is moderate (middle column), an increase in persuasiveness strongly increases the bias in the final decision. This result is explained by the same reason than before: a group with a highly talkative individual reach consensus because the influential individual convinces the rest of the group and pull their opinions toward its own. Finally, an increase in stubborness $\gamma_{L}$ has a limited effect in the bias of the decision, even when talkativeness and persuasiveness are low. This is because the group can still reach consensus even when one individual has an extreme opinion, and thus the presence of a stubborn individual does not pull the final decision toward an extreme.

## 4 Discussion

Consensus decision-making is a pervasive method for social groups to coordinate (Conradt and Roper, 2005). It has the benefit that it can be used to coordinate a wide range of collective tasks, unlike context specific coordination such as swarm movement. Yet, it can also carry costs. For instance, long time to reach consensus can lead to individuals abandoning the task for better alternatives (Skyrms, 2003) or even fission of the group (Krause and Ruxton, 2002). Leaders could limit this risk by speeding up the consensus decision-making. Yet, the absence of a mechanistic model of consensus decision-making has limited the investigation of the effect of leaders. To fill this gap, we used an opinion formation model which integrates heterogeneity in individuals' capacity to influence. We use numerical simulations to investigate the qualitative effects of number of leaders and their communication traits on the consensus time and the final decision.

First, our results show that the presence of influential leaders and influenceable followers reduces the time a group spend to reach consensus. In other words, the benefit of leadership on consensus decision-making can emerge from
the difference in individuals' capacity to influence others. This result is in line with previous work in game theory which shows that a dimorphism into leaderfollower behaviours could facilitate coordination (Johnstone and Manica, 2011), and shows that this conclusion can be extended to species using communication to coordinate rather than copying others' behaviour. Second, our results show that a single highly influential leader is the most efficient in term of consensus time, but that leaders with limited influence are preferred when multiple leaders are present. This suggests that social groups would favour strong leaders only in particular conditions, i.e. when they are able to enforce the presence of a single leader, such as in leadership based on conditional behaviours or by design e.g. institutional leadership (Perret et al., 2019). On the other side, the influence of leaders in many social organisms could be limited considering that multiple leaders are likely in nature because of the variations due to evolutionary processes.

Third, our results show that the presence of influential leaders and influenceable followers biases the decision toward the preferences of the leaders. This bias can ultimately affect the fitness of individuals as groups often have to decide between mutually exclusive activities and individuals differ in their preferences e.g. travel destination, type of food or timing (Conradt and Roper, 2005). This bias could also be detrimental as it limits the inflow of information from the followers. This can be harmful if followers possess knowledge that leaders lack (Koriat, 2012), or because followers often have more accurate knowledge by being closer to the ground (Ostrom, 1990). A promising development to study the cost of bias is through the use of information cascade models, which simulate how information is transmitted within a social network (Jalili and Perc, 2017).

Fourth, our results show that talkativeness is the crucial characteristic explaining the two effects of leaders on consensus decision-making: a reduction in time to consensus and a bias in final decision. In addition, our results show that the effect of persuasiveness of leaders is highly dependant of its talkativeness.

The work presented here shows that opinion formation models can provide a mechanistic model which describes the role of leadership in consensus decision-
making and that can be applied across a wide range of domains. Consensus decision-making has often been ignored or simplified in model of leadership in life sciences. For instance, previous model studying animal (Conradt and Roper, 2003) or human leadership (Powers and Lehmann, 2014) considered only despotic (one leader) or democratic (majority rule) groups. Yet, they are two extremes on a range of possible form of social organisation and a wide range of forms of leadership can be observed in nature (Von Rueden et al., 2014; Walker et al., 2017). This diversity can be integrated in opinion formation models, and allows a more thorough investigation of the evolution of leadership, as shown recently on the evolution of human leadership (Perret et al., 2020). The model presented here can similarly be tailored to investigate leadership in non-humans species that appear to use consensus processes to take collective decision e.g. bees, swans, wild dogs (Conradt and Roper, 2005).

This work expands on previous research in social dynamics. In particular, a previous opinion formation model investigated the effect of persuasiveness, stubbornness and talkativeness (called reputation in their models) on consensus decision-making (Gavrilets et al., 2016). However, this prior work have two differences with the model and analysis presented here. First, their mathematical approximation focus on the effect of population change in a single trait. For instance, they show that an increase in the mean persuasiveness of a group always reduces the time to consensus because individuals convince each other faster. We complete their work by looking at cases when these traits co-vary as observed in nature. We showed that in these conditions, consensus is reduced only when a minority of individuals are present and thus, we find back the benefit of leadership. Second, their simulations focus on the variability in the traits rather than their distribution. Thus, their results showing a benefit of leadership is limited to one of their shortest simulations being obtained when there was one persuasive, stubborn and talkative individual. Our findings confirm this result and provide a more thorough exploration. Finally, our results broaden their conclusion by showing that this effect is dependant of the number of leaders and the difference of influence between leaders and followers. In particular, we
show that multiple influential leaders can have a limited benefit, because leaders persuade each others' followers, creating conflict of interest between a large proportion of the group.

We considered here a complete network and only global consensus, i.e. all the group agree. Despite both being conservative assumptions, they are two unlikely features of real world situations. Jalili (2013) develops an continuous opinion formation considering local consensus and looked at the effect of the distribution of persuasion (called social power) within different network structures. This model shows that when persuasion is asymmetrically distributed with the most connected individuals having the highest social power, the consensus is largely improved with the largest cluster at the end of consensus moving from 30 to 85 percent of the total. Yet, this result does not hold on other network structures in which there is not large differences in number of social links. In brief, their results suggest that a minority of talkative and persuasive individuals also facilitate consensus decision-making when local consensus are considered. Further work could integrate network structure to investigate the effect of hierarchy and group size as defined here on the time to consensus. However, this requires a good representation of the social structure of individuals during consensus decision-making, which can be more dynamic than the social network observed in long-term interactions.

The model developed here predicts a relationship between the distribution of individuals' capacity to influence and the time that a group spend to reach consensus. Previous work (Kearns et al., 2009) has investigated how network structure and incentives affect human groups to reach a consensus before a given time limit using behavioural economics experiments. Their results support our predictions that groups with a minority of individuals with large influence (in their case, well-connected individuals) success more often to reach consensus. Our results also predict that (i) talkativeness is the most important characteristic of leaders and (ii) that persuasiveness is important when leaders are talkative. These predictions fit with experiments on human groups. First, it has been shown that most talkative individuals are recognised as leaders (the
"babble hypothesis") (Bass, 1949; Sorrentino and Boutillier, 1975). Second, this conclusion has been latter refined with experiments that show that quality of communication is also important but yet depends of the talkativeness of the individual communicating (Riecken, 1958; Jones and Kelly, 2007). More broadly, the difficulty of measuring the distribution of individual capacity to influence others has limited experimental measures. However, further test of our predictions could be done with developing methods to measure influence of individuals in animal groups (Strandburg-Peshkin et al., 2018; Richardson et al., 2018). Influence can also be measured a posteriori from transcript of human communication, where one can measure the impact of an individual's speech on the content of further communications (for instance, see Barron et al. 2018).

In conclusion, this model contributes to support the hypothesis that leadership provides a benefit to group organisation (Calvert, 1992). Our results complete this hypothesis by showing that the difference in individual capacity to influence is sufficient to explain the organisational benefit of social hierarchy. How much does this benefit, i.e. taking faster decisions, rather than a competency benefit, i.e. taking better decisions, explain the emergence of leaders? When faced with a task which can be solved by an optimal course of action, and given that competences are easy to assess, it is likely that the emergence of leaders would be driven by their capacities to take the right decision. (Gavrilets et al., 2016). This fits with the type of leadership observed in small-scale societies where skills are well-known by all (Garfield et al., 2019b). However, when the best solution for a task is not obvious or when there are multiple optimal solutions, when time is pressing, or when competences of individuals are hard to assess, the benefit brought by leaders on time to consensus could be the main driver behind the emergence of leadership. For instance, permanent and influential leaders are observed in large-scale human societies where group size limits the assessment of competences, the number of collective tasks can be very high, and tasks do not have an obvious solution (the payoff of a new rule regulating markets is hard to measure, for example). Promising future work would consist in adding concrete tasks to this decision-making model to better identify which
benefit is likely to drive the emergence of leadership. More broadly, merging the body of work on leadership in life sciences and opinion formation in physical sciences, could be a fertile ground for further research. We have shown here that opinion formation models can provide a in-depth description of the consensus decision-making and connect individual characteristics to group functioning. More than providing new understanding, these models also carry potential for managing group coordination. For instance, theoretical work have proposed algorithms to maximise the spread of information within groups (AskariSichani and Jalili, 2015). Similar work focusing on how bacteria regulate their virulence using collective decision-making by quorum sensing, could also provide new ways to control it (Rutherford and Bassler, 2012).

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[^0]:    ${ }^{1}$ Some models talk of reputation instead of persuasiveness (Gavrilets et al., 2016; Chen et al., 2016) but both are defined as the weight of the opinion of an individual on the opinion of someone else. This distinction is rather on the determinism of this feature, either being an intrinsic feature, persuasiveness or given by others, reputation

