A MODIFIED LOO MODEL WITH SECTORED AND THREE DIMENSIONAL MULTIPATH SCATTERING

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Abstract

In frequency non - selective mobile fading channels the energy can arrive at the mobile receiver via a three dimensional (3-D) scattering mechanism. That case occurs especially in urban environments, in which the tall buildings and other obstacles cause an arrival of multipath energy in the elevation plane, besides that arriving in the azimuth plane. Another issue, which is a matter of investigation, is that an important portion of multipath energy may not arrive at all in the mobile receiver, due to its blocking by the shadowing mechanisms of the channel. In this work we propose a model which takes into account both 3-D multipath scattering and partial arrival of multipath energy due to shadowing. Moreover a line of sight (LOS) component exists, which is also influenced by the shadowing.

I. Introduction

The transmission performance of wireless services is strongly influenced by the rapid amplitude and phase fluctuations of the received signal. Those fluctuations result from the constructive and destructive nature of the arriving multipath components at the receiver. Moreover an important contribution to the above mentioned fluctuations arises from the time varying attenuation of the received signal mean value due to shadowing.

In order to model the slow term variations due to shadow fading and incorporate them in the rapid short term variations, arising from multipath propagation, two basic models have been proposed. Each of them represents a different concept for the wireless mobile channel modeling. The first one was proposed by Suzuki [1] and Hansen and Meno [2], the so called Suzuki process. This model is obtained by multiplying a Rayleigh process with a lognormal process. The second one was proposed by Loo [3]. This model resembles a Rician model, with the additional property that the LOS component is no more constant, as this happens in Rice probability density function (PDF), but it is a random stochastic process following a lognormal PDF. Loo's model arises by summing a lognormally distributed random phasor and a Rayleigh phasor.

In international bibliography the term "modified" applies to the case where the inphase and quadrature Gaussian components generating the Rayleigh part are correlated, whereas the term "extended" refers to the case where the Rayleigh component has been substituted by a Rician one. Thus we obtain modified Suzuki processes, extended Suzuki processes and modified Loo models. Several modifications and extensions can be found in literature for both models, with the most intuitive and analytical presentations given in [4 - Ch. 6]. By adopting modified models we force the Doppler power spectral density (PSD) to obtain an asymmetrical shape, in contrast to the classical symmetrical two dimensional (2-d) Doppler PSD given by Clarke [5]. Thus, it is a simple technique to have a sectored arrival of multipath energy, when a part of it is blocked by the channel obstacles, or directional antennas are used.

In this work we propose a modified version of Loo model which takes into account both 3-D multipath scattering and sectored arrival of multipath energy. To do so we employ for each Gaussian component of the Rayleigh part, a PSD similar to that proposed by Aulin [6] and a cross correlation scheme between the two components similar to that proposed by Patzold et all [7]. Moreover the LOS component is Doppler shifted due to the receiver's mobility, as considered in [7] too.

This paper is organized as follows. Section II gives the analytical model for the mobile channel. More specifically the Doppler PSD and the PDF of the amplitude are derived. Afterwards the second order statistics, level crossing rate (LCR) and average duration of fades (ADF's), are investigated. In Section III we demonstrate the flexibility and usefulness of the model by adapting the second order statistics (LCR) to real world channels drawn from measurements. Finally Section IV concludes this paper with a synopsis of the main results and advantages of the model.

II. The analytical model

A suitable complex stochastic process $\mu_\rho(t)$ for the wireless mobile channel arises by summing a lognormally distributed random phasor and a Rayleigh phasor [3] and using the notation reported in [7], we obtain

$$\mu_\rho(t) = \rho(t)\exp[j(2\pi f_\rho t + \theta_\rho)] + \mu(t) \qquad (1)$$

where t is the time parameter, $\rho(t)$ is a lognormal process denoting the time varying amplitude of the LOS component and f_ρ and θ_ρ are its Doppler frequency and phase respectively. The process $\mu(t)$ is a complex Gaussian one, with its amplitude being Rayleigh distributed. Those processes are derived by the real, zero mean Gaussian processes $\nu_1(t)$ and $\nu_2(t)$ as follows

$$\mu(t) = \nu_1(t) + j\nu_{1h}(t) \qquad (2)$$

$$\rho(t) = \exp(s\nu_2(t) + m) \qquad (3)$$

where $\nu_{1h}(t)$ denotes the Hilbert transform of $\nu_1(t)$ and $\nu_2(t)$ has unit variance. The parameters s and m are characteristic quantities of the shadowing environment. From the above notation it is clear that the inphase and quadrature components of the complex Gaussian process $\mu(t)$ are correlated. Finally the stochastic process $r(t)$ describing the wireless channel is obtained by taking the absolute value of the left side in equation (1). By doing this, we have

$$r(t) = |\mu_\rho(t)| = \sqrt{\mu_{\rho 1}(t)^2 + \mu_{\rho 2}(t)^2} \qquad (4)$$

where

$$\mu_{\rho 1}(t) = \nu_1(t) + \rho(t)\cos(2\pi f_\rho t + \theta_\rho) \qquad (5)$$

$$\mu_{\rho 2}(t) = \nu_{1h}(t) + \rho(t)\sin(2\pi f_\rho t + \theta_\rho)\,. \qquad (6)$$

The stochastic process $r(t)$ is an appropriate model to describe narrowband wireless mobile fading channels in the complex baseband. Additionally by choosing appropriate PSD shapes for the multipath mechanism we can model 3-d multipath scattering.

A. The Doppler PSD shape

In order to account for the Doppler PSD, taking into consideration 3-D scattering, we make use for the process $\nu_1(t)$, the following form proposed in [6]

$$S_{\nu_1\nu_1}(f) = \begin{cases} \dfrac{\sigma_0^2}{2\pi f_{\max}\sin b_m}\left(\dfrac{\pi}{2} - \arcsin\dfrac{2\cos^2 b_m - 1 - (f/f_{\max})^2}{1-(f/f_{\max})^2}\right), |f| < f_{\max}\cos b_m \\ \dfrac{\sigma_0^2}{2 f_{\max}\sin b_m}, f_{\max}\cos b_m \le |f| \le f_{\max} \\ 0, |f| > f_{\max} \end{cases} \qquad (7)$$

where $S_{\nu_1\nu_1}(f)$ is the PSD of $\nu_1(t)$ and $f_{\max}$ the maximum Doppler frequency, equals to uf_0/c, with u the mobile receiver's velocity, c the speed of light in free space and f_0 the carrier frequency. The parameter σ_0 determines the mean power of $\nu_1(t)$ and b_m is the maximum elevation angle relative to the azimuth plane in which the receiver moves [6], $0 \le b_m \le \pi/2$. The above equation (7) if normalized such that $\int_{-\infty}^{\infty} S_{\nu_1\nu_1}(f)df = 1$, constitutes a PDF for the Doppler frequencies. The spectrum in (7) arises by considering an angle of arrival PDF of β $p_\beta(.)$, in the elevation plane as [6]:

$$p_\beta(\beta) = \begin{cases} \dfrac{\cos\beta}{2\sin b_m} & , |\beta| \le b_m \le \dfrac{\pi}{2} \\ 0 & , otherwise \end{cases} \qquad (8)$$

where β is the elevation angle relative to the azimuth plane, in which the receiver moves (see figure 1). If $b_m \to 0$ the scattering occurs explicitly in the azimuth plane and the spectrum in (7) tends to the classical U-shaped spectrum given in [5].

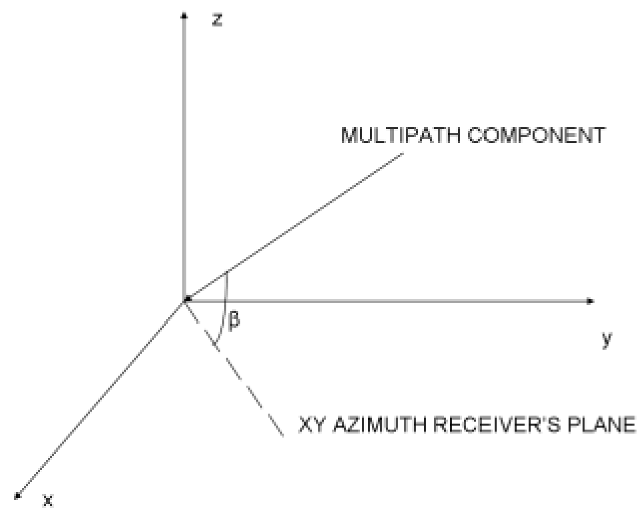

Fig.1. Elevation angle of arrival β of a multipath component.

The inverse Fourier transform of (7) determines the autocorrelation function $r_{\nu_1\nu_1}(\tau)$ of the process $\nu_1(t)$, with τ the difference between two time instants, defined in [6] as

$$r_{\nu_1\nu_1}(\tau) = \sigma_0{}^2 \int_{-b_m}^{b_m} J_0(2\pi f_{\max}\tau\cos\beta)\, p_\beta(\beta)\, d\beta \qquad (9)$$

with $J_0(.)$ the zeroth-order Bessel function of the first kind. The cross correlation function $r_{\nu_1\nu_{1h}}(\tau)$ between the processes $\nu_1(t)$ and $\nu_{1h}(t)$ is obtained by employing the equivalent scheme of (9) which, taking into account that $S_{\nu_1\nu_1}(f)$ is an even function, arises as

$$r_{\nu_1\nu_1}(\tau) = 2\int_0^{f_{\max}} S_{\nu_1\nu_1}(f)\cos(2\pi f\tau)df \ . \qquad (10)$$

Thus $r_{\nu_1\nu_{1h}}(\tau)$ is obtained by a phase shift of $-\pi/2$ in (10), giving

$$r_{\nu_1\nu_{1h}}(\tau) = 2\int_0^{f_{\max}} S_{\nu_1\nu_1}(f)\sin(2\pi f\tau)df \ . \qquad (11)$$

The Doppler PSD of the process $\nu_2(t)$ is assumed to be a Gaussian function. A symmetrical spectrum around zero frequency is appropriate to model the time selectivity of the LOS component in a local area, as that probably results from the scatterer's mobility. Thus

$$S_{\nu_2\nu_2}(f) = \frac{1}{\sqrt{2\pi}\sigma_c}\exp\left(-\frac{f^2}{2\sigma_c^{\ 2}}\right) \qquad (12)$$

where σ_c is related to the 3-dB cut-off frequency f_c, according to $f_c = \sigma_c\sqrt{2\ln 2}$.

The Doppler PSD for the lognormal process $\rho(t)$ is given in [4] as:

$$S_{\rho\rho}(f) = \exp(2m+s^2)\times$$
$$\left\{\delta(f) + s^2 S_{\nu_2\nu_2}(f) + \sum_{n=2}^{\infty}\left[\frac{s^{2n}}{n!}\cdot\frac{S_{\nu_2\nu_2}\left(f/\sqrt{n}\right)}{\sqrt{n}}\right]\right\} \ (13)$$

where $\delta(.)$ is the Dirac delta function. Finally the sectored Doppler PSD for the complex stochastic process $\mu_\rho(t)$ is given in [7] as:

$$S_{\mu_\rho\mu_\rho}(f) = 2[1+sign(f)]S_{\nu_1\nu_1}(f) + S_{\rho\rho}(f-f_\rho) \ (14)$$

An example of the PSD function $S_{\mu_\rho\mu_\rho}(f)$ for a parameter set chosen as $f_{\max} = 91Hz$, $\sigma_0 = 1$, $s = 0.3$, $m = 0.5$, $f_\rho = 0.3 f_{\max}$, $f_c = 8Hz$ and $b_m = \pi/6$ is given in figure 2.

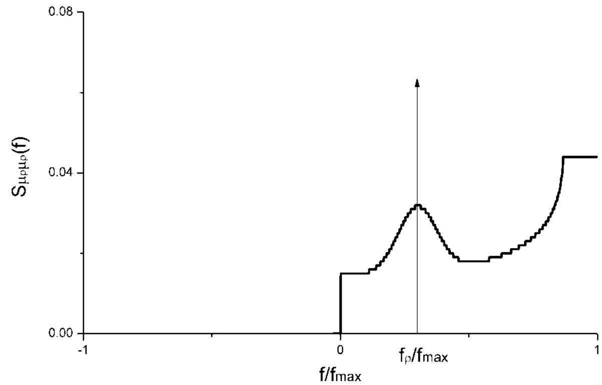

Fig.2. Doppler PSD function of the process $S_{\mu_\rho\mu_\rho}(f)$.

Before we proceed it is pertinent to define the following parameter set, convenient for the rest of the paper. More specifically we have

$$\psi_0 = r_{\nu_1\nu_1}(0) = r_{\nu_{1h}\nu_{1h}}(0) \qquad (15)$$

$$\phi_{01} = r'_{\nu_1\nu_{1h}}(0) \qquad (16)$$

$$\psi_{02} = r''_{\nu_1\nu_1}(0) = r''_{\nu_{1h}\nu_{1h}}(0) \qquad (17)$$

where the primes denote derivatives with respect to the time difference τ. The parameter ψ_0 is the mean power of the processes $\nu_1(t)$ and $\nu_{1h}(t)$. By using equation (9) in (15) and (17), equation (11) in (16), after some algebraic manipulations and taking into account a modified version of equation (2.835) in [8] we obtain

$$\psi_0 = \sigma_0^{\ 2} \qquad (18)$$

$$\phi_{01} = 2\sigma_0^{\ 2} f_{\max}\left[\cos b_m + \left(b_m/\sin b_m\right)\right] \qquad (19)$$

$$\psi_{02} = -\pi^2 f_{\max}^{\ \ 2}\sigma_0^{\ 2}\left[\cos\left(2b_m\right)+5\right]\Big/3. \qquad (20)$$

Moreover the parameters a and b should be defined as

$$b = -\psi_{02} - \phi_{01}^{\ 2}/\psi_0 \qquad (21)$$

$$a = \left[2\pi f_\rho - (\phi_{01}/\psi_0)\right]\Big/\sqrt{2b} \ . \qquad (22)$$

B. *PDF of amplitude, LCR and ADF's*

The PDF of the amplitude $p_r(z)$ is similar to that given in [3] and [7]. Thus

$$p_r(z) = \frac{z}{\sqrt{2\pi}s\psi_0}\int_0^\infty \frac{1}{y} I_0\left(\frac{zy}{\psi_0}\right)\times$$

$$\exp\left[-\frac{(\ln y-m)^2}{2s^2}\right]\exp\left(-\frac{z^2+y^2}{2\psi_0}\right)dy,\ z\geq 0 \quad (23)$$

with $I_0(.)$ the modified Bessel function of zeroth order. As it can be seen from equation (23) the amplitude PDF is independent from the maximum elevation angle b_m in which the scattering occurs. It is dependant on the parameters s, m and σ_0. For the special case of $s=0$ and $m=\ln\rho$ the lognormal process becomes $\rho(t)=\rho$ and equation (23) becomes the Rician PDF as follows

$$p_r[z/\rho(t)=\rho]=\frac{z}{\psi_0}\exp\left(-\frac{z^2+\rho^2}{2\psi_0}\right)I_0\left(\frac{z\rho}{\psi_0}\right). \quad (24)$$

The amplitude PDF is depicted graphically in figure 3 as a function of σ_0. The remaining parameters are $s=0.3$ and $m=0.5$.

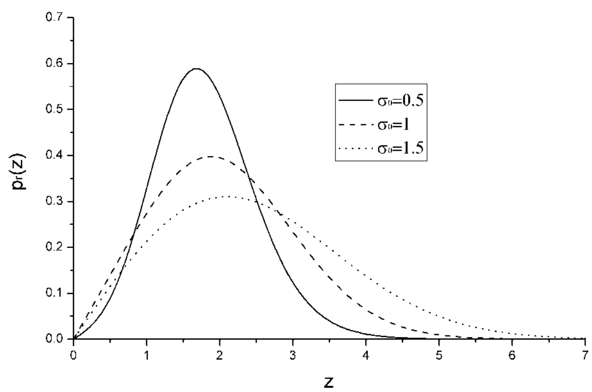

Fig.3. The amplitude PDF as function of σ_0.

The expression for the LCR $N_r(z)$, i.e. the average number of crossings per second at which $r(t)$ crosses a specified signal level z with positive slope is similar to that given in [7], but with different characteristic quantities ψ_0, ϕ_{01}, ψ_{02}, a and b defined in equations [(18)-(22)]. Thus

$$N_r(z)=\int_0^\infty N_{r\rho}(z,y)\,p_\rho(y)dy,\ z\geq 0 \quad (25)$$

where $N_{r\rho}(z,y)$ is the LCR when the LOS component is not lognormally distributed and has constant amplitude equals to $y\geq 0$ and $p_\rho(y)$ is the lognormal PDF for the LOS component, both defined as [7]

$$N_{r\rho}(z,y)=\frac{z\sqrt{2b}}{\pi^{3/2}\psi_0}\exp\left(-\frac{z^2+y^2}{2\psi_0}\right)\int_0^{\pi/2}\cosh\left(\frac{zy\cos\theta}{\psi_0}\right)\times$$
$$\left\{\exp[-(ay\sin\theta)^2]+\sqrt{\pi}ay\sin\theta\cdot erf\left(ay\sin\theta\right)\right\}d\theta,$$
$$z\geq 0,\ y\geq 0 \quad (26)$$

$$p_\rho(y)=\frac{1}{\sqrt{2\pi}sy}\exp\left[-\frac{(\ln y-m)^2}{2s^2}\right],\ y\geq 0. \quad (27)$$

Equations (25) and (26) are holding if $f_{\max}/f_c\geq 5$ as demonstrated in [7], i.e., if the amplitude of the LOS component changes relatively slowly in time compared with the rate of change of the diffuse component. The last assumption is valid in real world channels, where $f_{\max}/f_c>>1$ usually holds. From equations (25), (26) and (27) it is clear that the LCR is proportional to the maximum Doppler frequency $f_{\max}$.

The ADF's $T_r(z)$, i.e. the mean value of the time intervals at which $r(t)$ remains below a specified signal level z is given by the well known equation

$$T_r(z)=\frac{F_r(z)}{N_r(z)} \quad (28)$$

where $F_r(z)$ is the cumulative distribution function of the process $r(t)$, defined as

$$F_r(z)=\int_0^z p_r(x)dx. \quad (29)$$

From equations (28) and (29) we see that the ADF's is inverse proportional to $f_{\max}$.

If $b_m\to 0$ the expressions for the PDF, LCR and ADF's are the same with those in [7], considering a Clarke spectrum for $\nu_1(t)$ there ($k_0=1$ in [7]). The normalized LCR $N_r(z)/f_{\max}$ and normalized ADF's $T_r(z)\cdot f_{\max}$ are depicted graphically as a function of b_m in figures 4 and 5 respectively. The remaining parameters for both figures are the same as in figure 2.

It is clear from figure 4 that with increased elevation angle of arrival (b_m increases) the LCR decreases because the multipath propagation reduces its influence as projected to the receiver's azimuth plane. Thus fluctuations occur less frequently. Moreover from figure 5 with increased elevation angle of arrival the ADF's increases, meaning more time the signal remains below small, medium and large levels, or equivalently fluctuates less frequently.

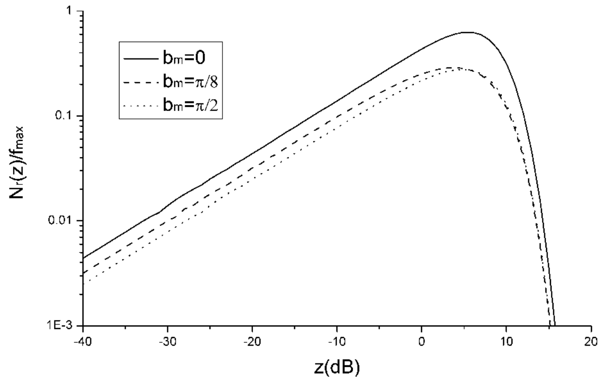

Fig.4. The normalized LCR as a function of b_m.

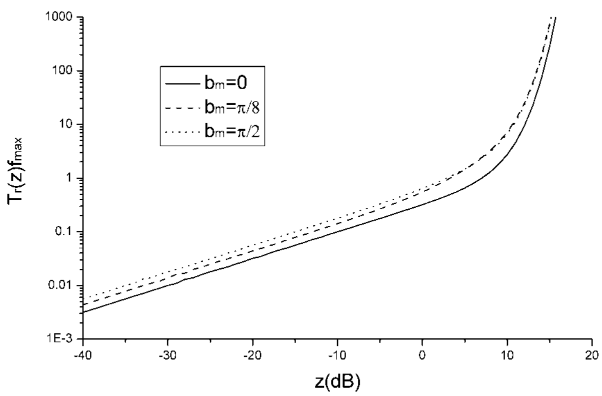

Fig.5. The normalized ADF's as a function of b_m.

III. Applications to real world channels

In this section we demonstrate the flexibility and usefulness of the proposed model by adapting its second order statistics (LCR) to data drawn from measurements. More specifically we consider the measurements of the LCR in [9]. The environments studied there were, the one a rural area with almost 35% tree cover (heavy shadowing) and the other an open area with almost no shadowing (light shadowing).

Our task is to find the proper values of the model parameters (s, m, b_m, σ_0), such that the absolute value of the difference between the analytical and measured LCR's is minimum (ideally zero). In order to do so we combine the model parameters to a multi-parametric function, seeking its minimum. That function is

$$g(s,m,b_m,\sigma_0)=\frac{1}{f_{\max}}\left(\sum_{n=1}^{N}\left[\left(\frac{N_r(z_n)-N_m(z_n)}{N_m(z_n)}\right)^2\right]\right)^{1/2} \quad (30)$$

where $N_m(.)$ is the measured LCR and N the number of measured values. The minimization of (30) is carried out by applying any method of optimization inherent in mathematical software packets. By doing this, we find for the two cases (light and heavy shadowing) the parameter set given in table I.

The maximum Doppler frequency $f_{\max}$ is kept constant to 91Hz and is not optimized, as it does not constitute a channel parameter, being depended on the mobile unit speed and carrier frequency. The same holds for the Doppler frequency of the LOS component, as it is a rational assumption that in a local area that component will have a specific angle of arrival relative to the receiver's motion. Thus we arbitrarily set $f_\rho = 0.5 f_{\max}$. The remaining parameters of the model (f_c and θ_ρ) which do not affect the normalized LCR are arbitrarily set as $f_c = 8Hz$ and $\theta_\rho = 0$. Thus, all parameters of the model are determined and the resulting analytical and measured normalized LCR's, $N_r(z)/f_{\max}$ and $N_m(z)/f_{\max}$ respectively, are shown in figures 6 and 7 for the two cases.

The light shadowing case reveals a fairly good agreement between the proposed model and the measurements data. On the contrary the agreement, in the heavy shadowing case, is much better. The reason for this is that the PSD we employ here [eq. (14)] is suitable to model environments with 3-D multipath scattering, due to the interaction of electromagnetic waves with tall objects an assumption which much better fulfilled in heavy shadowing environments than open areas with almost no shadowing.

We should also notice how the parameter b_m characterizes each environment. In heavy shadowing, $b_m = \pi / 2.83 = 63.604^\circ$, which is a rational result, as most likely the diffuse component will arrive at the receiver after interacting with tall objects. On the contrary, for the light shadowing $b_m = 0$, which also seems rational, as most probably the scattering will explicitly occur in the azimuth receiver's plane.

TABLE I
MODEL PARAMETERS FOR LIGHT AND HEAVY SHADOWING

SHADOWING	s	m	b_m	σ_0
LIGHT	0	0.032	0	0.307
HEAVY	0.564	-1.817	π/2.83	0.228

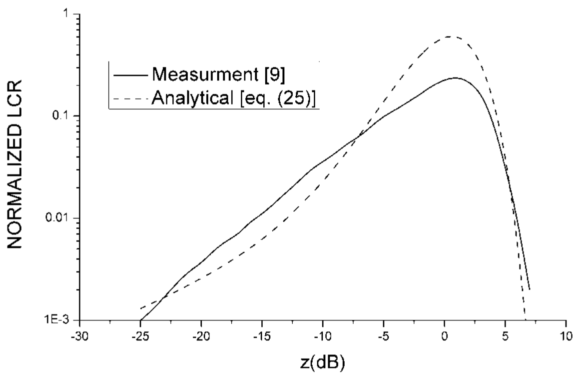

Fig.6. Normalized LCR for the light shadowing environment.

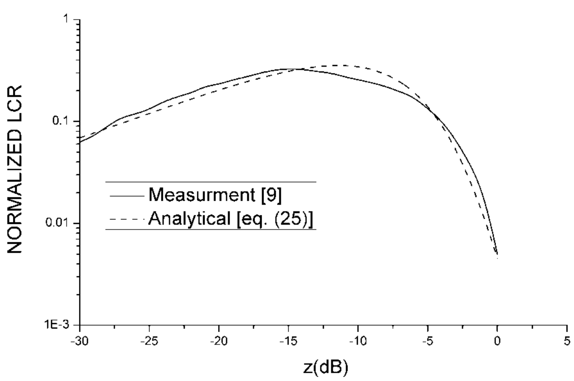

Fig.7. Normalized LCR for the heavy shadowing environment.

IV. Conclusions

A new modified Loo model which incorporates 3-d scattering was presented. It was shown that with increased elevation angle of arrival the received signal fluctuates less frequently. A curve fitting of the second order statistics (LCR) to real word data, drawn from measurements, validated the usefulness and flexibility of the proposed model. In the light shadowing environment a fairly good agreement between the proposed model and the measurements was revealed, whereas in the heavy shadowing the agreement was much better. This is justified because of the physical basis of our model. Moreover in open areas, with no obstruction of the LOS component, is expected that the multipath energy spreading will be more directional in the azimuth receiver's plane as compared with heavy shadowing cases. This is caused by the unsymmetrical positioning of the scattering objects around the receiver (ideally concentrated in special azimuth angles). The last directly leads to a less Doppler frequency spreading in open areas, compared with that in heavy shadowed areas. But that case is not predicted by our model which assumes equal Doppler frequency spreading in any case. A model with a variant Doppler frequency spreading, incorporating 3-D scattering, would be more general and would probably provide a better fitting to measurements data. The last is an open research issue, concerning the research activity of the group.

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