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The relationship between the golf swing plane and ball impact characteristics using
trajectory ellipse fitting
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#### Abstract

The trajectory of the clubhead close to ball impact during the golf swing has previously been shown to be planar. However, the relationship between the plane orientation and the orientation characteristics of the clubhead at ball impact has yet to be defined. Fifty-two male golfers (27 high skilled, 25 intermediate skilled) hit 40 drives each in an indoor biomechanics laboratory. This study successfully fitted the trajectory of the clubhead near impact to an ellipse for each swing for players of different skill levels to help better explain this relationship. Additionally, the eccentricities of the ellipses were investigated for links to skill level. The trajectory of the clubhead was found to fit to an ellipse with RMSE of 1.2 mm . The eccentricity of the ellipse was found to be greater in the high skilled golfers. The club path and angle of attack generated from the ellipse fitted clubhead trajectory were found to have a normalised bias-corrected RMSE of $2 \%$ and $3 \%$ respectively. A set of 'rule of thumb' values for the relationship between the club path, angle of attack and delivery plane angle was generated for use by coaches.


Keywords: Plane fitting, trajectory, eccentricity, striking, performance

## 1. Introduction

Analysis of golf swing technique promoted by the Professional Golfers Associations (PGA) of the UK and USA appears to broadly follow a deterministic model (PGA, 2012; Wiren, 1991). They suggest that changes should only be made to the swing technique if it has a direct influence on the impact characteristics of the golf shot, and, consequently the flight and
outcome of the shot. The relationships between ball launch variables and clubhead impact characteristics have been identified and give validity to this model (Betzler, Monk, Wallace, \& Otto, 2014; Sweeney, Mills, Alderson, \& Elliott, 2013). There has also been some investigation into the relationship between technique during the complete swing and impact characteristics (Brown et al., 2011; Chu, Sell, \& Lephart, 2010; Joyce, Burnett, Cochrane, \& Ball, 2012; Sinclair, Currigan, Fewtrell, \& Taylor, 2014). However, the majority of this research is directed primarily at clubhead speed as an outcome variable, with very little research aimed at specific impact characteristics such as the club path and angle of attack (Keogh \& Hume, 2012).

One element of swing technique thought to have an influence on the club path at impact is swing plane (PGA, 2012; Wiren, 1991). Jenkins (2007) dates the concept back to the turn of $20^{\text {th }}$ century with Seymour Dunn and his description of an elliptical path on an oblique plane. With the clubhead trajectory modelled as an ellipse, ball strikes earlier or later on this arc will have related effects on the path and angle of attack of the club as it strikes the ball. Combined with the orientation of the plane on which the ellipse sits the relationship between club path and swing plane may well be simply geometrical.

Although the swing plane has been modelled in many different ways (Coleman \& Anderson, 2007; Hardy \& Andrisani, 2005; Hogan, 1957; MacKenzie, 2012), recent studies have returned to this concept of the clubhead trajectory near impact being on an inclined plane (Kwon, Como, Singhal, Lee, \& Han, 2012; Morrison, McGrath, \& Wallace, 2014). While portions of the swing near impact have been shown to be highly planar, the trajectory has yet to be shown to follow an ellipse nor has the relationship with the club path and angle of attack been validated.

Another key consideration of an elliptical trajectory would be its shape, or eccentricity. If the clubhead does travel on an elliptical path, the rate of change of the gradient of the ellipse, and thus the clubhead trajectory, would be lowest when the clubhead is travelling close to parallel with the long radius of the ellipse. This rate of change would be lower again if the ellipse were more eccentric. With a reduced rate of change of the clubhead trajectory, it is hypothesised here that any change in the position of the low point of the arc relative to the ball position will have less of an effect on the club path and angle of attack at impact.

Variability in club path and the angle of attack have been shown to be important with respect to the variability in the shot outcome (Betzler et al., 2014) and skill level (Betzler, Monk, Wallace, \& Otto, 2012). The mechanism by which high skilled golfers reduce this variability is a valid line of investigation with the shape of the clubhead trajectory potentially yielding important insights.

Consequently, the primary aim of the present study was to determine how well the trajectory of the club near impact fitted to an ellipse on an inclined plane, including how this and the orientation of the plane differed between skill levels. The eccentricity of the ellipse was also investigated in relation to skill level, with a research hypothesis that the fitted ellipses would be more eccentric in the high skilled golfers.

## 2. Methods

### 2.1. Participants

Fifty-two male injury-free golfers were recruited from two skill levels: 27 high skilled golfers with CONGU handicaps of 5 and below (mean $\pm$ SD: age $25.5 \pm 7.5 \mathrm{yr}$; mass $79.5 \pm 11.5 \mathrm{~kg}$; height $1.82 \pm 0.37 \mathrm{~m}$; handicap $0.6 \pm 2.8$ ), and 25 intermediate skilled golfers with handicaps ranging from 10-18 (age $39.4 \pm 11.2 \mathrm{yr}$; mass $87.1 \pm 11.3 \mathrm{~kg}$; height $1.80 \pm 0.65 \mathrm{~m}$; handicap $13.2 \pm 2.8$ ). The study was approved by the University's Research Ethics committee with all
participants providing written informed consent, and conforms to the requirements stipulated in the Declaration of Helsinki.

### 2.2. Procedure

### 2.2.1. Apparatus

A 12-camera, 1000 Hz Oqus 300 system and Qualisys Track Manager (Qualisys AB, Gothenburg, Sweden) were used to collect and calculate three-dimensional coordinate data. Three spherical retro-reflective markers each with a diameter of 12.7 mm were attached to the crown of the club, and two pieces of retro-reflective tape were attached to the shaft just below the grip and a further 20 cm below that for dynamic tracking. Five 6.4 mm diameter markers were attached to the clubface (figure 1), and removed after static capture. The ball position was defined by a small piece of unobtrusive retro-reflective tape attached to the top of the golf ball. During processing this point was translated vertically downwards by the radius of the ball and thus represented the centre of the golf ball. A similar marker set has been used previously and validated by Betzler et al. (2012).

Figure 1. Clubhead marker setup. Face markers were placed on the top and bottom grooves of the toe and heel. The centre marker is located in the geometric centre of the clubface


Each golfer used their own driver with which they were familiar. Whilst the clubhead markers added 10 g to the mass of the club, this mass adjustment has not been shown to be reliably detected by golfers and has little effect on shot performance (Harper, Roberts, \& Jones, 2005). No negative consequences of marker attachment were reported by the players in the present study.

### 2.2.2. Equipment setup

The testing took place in an indoor biomechanics laboratory. Participants hit shots from a golf mat into a net situated 10 m away. A fairway and target were projected onto the net to increase the ecological validity of the setup. Prior to commencing the 40 shots, the players were shown the target and asked to hit the longest drives they felt comfortable hitting while still keeping the ball on the projected fairway.

### 2.2.3. Data collection

Following a self-directed warm up hitting shots, a static file was captured from which to later build the model in Matlab (R2014a, The Mathworks, Inc., Natick, MA, USA). Forty golf shots were captured for each player, regardless of the quality of the shot outcome (this included all shots where the face of the club made contact with the ball). Players were instructed to attempt the same type of shot each time to avoid multiple shot strategies being used. To prevent fatigue effects, a minimum of 45 s delay between shots was enforced and a 5-min break after every 8 shots was imposed.
2.3. Data analysis

### 2.3.1. Data reduction

Data analysis was carried out using Matlab. The clubhead model was based on that of Betzler et al. (2012), which has previously been validated. The face markers were fitted to a sphere of
radius 253 mm , and then translated 3 mm back onto the club face. The instant of impact between club and ball was often not captured, even at a capture frequency of 1000 Hz . The last frame in which the centre of the club head sphere and the centre of the ball were further apart than their combined radii was taken as initial impact, and all post-impact data were subsequently removed.

As the data up to impact were used in the analysis, data padding was used when filtering. Twenty data points were added using linear extrapolation before filtering, and then removed afterward (Giakas, Baltzopoulos, \& Bartlett, 1997; Vint \& Hinrichs, 1996). The data were filtered using a zero-lag $4^{\text {th }}$ order Butterworth filter (Brown, Selbie, \& Wallace, 2013; Horan \& Kavanagh, 2012; Kwon et al., 2012; Sinclair et al., 2014; Tucker, Anderson, \& Kenny, 2013). A cut-off frequency of 40 Hz was calculated using residual analysis (Winter, 2009). The start of the trial was also trimmed to the mid-downswing event; defined as the instant at which the two shaft markers were horizontal during the downswing.

### 2.3.2. Swing plane

As per Morrison et al. (2014), a plane, defined as the delivery plane, was fitted to the trajectory of the clubface centre from mid-downswing to impact using a least squares orthogonal distance fitting method. This delivery plane was then projected onto the xy and yz references planes. The angles of these projections to the $x$-axis and $y$-axis represented the horizontal plane angle and vertical plane angle respectively, where the x -axis was parallel to the ball to target line and the z -axis was vertically up (figure 2 ).

For each shot, the clubface centre trajectory from mid-downswing to impact was projected onto the delivery plane and subsequently fitted, via a least squares method, to an ellipse of the form:

$$
\frac{\left(x^{\prime} \cos \theta-y^{\prime} \sin \theta\right)^{2}}{a^{2}}+\frac{\left(x^{\prime} \sin \theta+y^{\prime} \cos \theta\right)^{2}}{b^{2}}=1
$$

where $x^{\prime}$ and $y^{\prime}$ are the coordinates of the points on ellipse after the rotation of the delivery plane, $a$ and $b$ are the long and short radii of the ellipse respectively, and $\Theta$ is the angle of the long radius to the x '-axis (also see figure 2) (Zatsiorsky, 2002).

A measure known as flattening (f) was used to represent the eccentricity of the ellipse (Burkholder, 1995). The measure gives the difference between major and minor radii over the major radii, presented as a percentage (equation 2), i.e. the percentage the short radius had decreased from being a circle:

$$
f=\frac{a-b}{a} \times 100
$$

where $a$ and $b$ are the long and short radii of the ellipse respectively, and $f$ is flattening.

Figure 2. Horizontal plane angle (HPA) and vertical plane angles (VPA) of the fitted plane, along with angle of rotation of the fitted ellipse ( $\Theta$ ). Dashed arc represents the original trajectory of the clubhead. The long and short radii of the ellipse are labelled $a$ and $b$ respectively. The $x$-axis was parallel to the ball-totarget line


### 2.3.3. Impact characteristics

Impact characteristics were calculated using a purpose-built Matlab based executable (Betzler et al., 2012, 2014). To avoid any distortion of the trajectories at the end point (impact) unfiltered data were used to calculate the impact characteristics. As the last frame before impact was not the first contact between club and ball, cubic extrapolation was used to determine the time at which this occurred. The horizontal and vertical directions of travel of
the face centre (club path and angle of attack respectively) were calculated for the last 10 frames before impact. Linear extrapolation was then used to find the values of club path and angle of attack at first contact with the ball. The same process was carried out to calculate the angle of attack and club path at the time of first contact with the ball for the ellipse fitted trajectory.

### 2.3.4. Ground strike detection

When striking a golf ball, the club occasionally hits the ground before the ball. With the ball elevated on a tee this does not always have a detrimental effect on the shot. As the present study investigated the shape of the clubhead trajectory, a collision that occurs during the delivery phase may have had an impact on the ellipse and plane fitting.

With a total of 2,080 shots collected an automated method for detecting a ground strike was devised. A straight line was fitted to the clubhead speed for last 10 frames for each shot; the median slope of the lines was then calculated for the 40 shots. Median was used as mean values would be skewed by the outlier being predominantly negative. Using the median slope value and the data point 10 frames pre-impact, an impact value was predicted. A threshold value of $0.75 \mathrm{~m} / \mathrm{s}$ was used for the difference between the actual and predicted impact values that separated the ground strikes with the clean strikes. In pilot testing this proved to be $100 \%$ accurate. Any shots not fulfilling this were removed.

From the 52 players, 2,080 golf shots were recorded of which 67 were deemed to have been ground strikes. Therefore, these were eliminated from the analysis. The most shots removed for one player was 18 .
2.4. Statistical analysis

Root mean square error (RMSE) was used to assess the fit of the trajectory to the plane for each swing (Kwon et al., 2012; Morrison et al., 2014), and also the fit of the ellipse.

The error in the ellipse fitted impact characteristics was assessed using RMSE; however, RMSE calculation assumes that there is no bias between the two measures (Chai \& Draxler, 2014). Therefore, prior to the RMSE calculations, ANOVA was used to assess whether the means of the ellipse fitted impact characteristics and the those calculated from the original data were significantly different. If no significant difference existed, then RMSE was calculated; however, if there was a significant difference then the bias was removed from the ellipse fitted data before calculating the RMSE (Chai \& Draxler, 2014). This was achieved by subtracting the difference between the means of the ellipse fitted and original impact characteristics from the ellipse fitted impact characteristics. RMSE was also normalised to the range of the data to give context to the error.

Figure 3. Example plot of actual (dashed line) and ellipse fitted (solid line) trajectory viewed in the $x-z$ plane.


Significances of between group differences were calculated for club paths, angles of attack, horizontal plane angle, vertical plane angle and ellipse flattening. Due to the number of dependent variables a MANOVA was initially implemented. Assuming the MANOVA showed a significant effect of skill level, ANOVA was used to compare the means for the variables meeting the parametric criteria. However, the flattening of the ellipse was found not to be normally distributed using the Kolmogorov-Smirnov test; therefore, the Mann-Whitney U test was used. The alpha level was set to 0.05 , and all statistical analyses were carried out using SPSS (Release 22, IBM).

## 3. Results

### 3.1. Delivery plane and ellipse fitting

The fit of the delivery plane was found to have a mean RMSE of 1.1 mm . The fit of the clubhead trajectory to the ellipse was found to have a mean RMSE of 1.2 mm . For individual players the RMSE ranges from 0.15 mm to 1.82 mm for club path and from 0.34 mm to 1.58 mm for angle of attack (figure 4).

The means of the ellipse fitted path and angle of attack were found to be significantly different from those calculated from the original data ( $\mathrm{p}<0.05$ ) (table 1 ). The ellipse fitted path was found to overestimate by $0.70^{\circ}$, while the ellipse fitted angle of attack was found to overestimate by $0.67^{\circ}$. Therefore, the normalised bias-corrected RMSE was found to $2 \%$ for the club path and 3\% for the angle of attack (table 2).

Table 1. Group means, standard deviations and statistical differences of club paths and angles of attack for actual and ellipse fitted trajectories (* denotes significant difference between skill levels ( $\mathrm{p}<0.05$ ), $\dagger$ denotes significant difference between ellipse fitted and actual impact characteristics)

|  | All players |  |  | High Skilled Group |  |  | Intermediate Skilled Group |  |  | Effect size of group diff. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean |  | SD | Mean |  | SD | Mean |  | SD | F | r |  |
| Path Actual ( ${ }^{\circ}$ ) | -2.1 | $\pm$ | 4.0 | -0.8 | $\pm$ | 2.6 | -3.5 | $\pm$ | 4.7 | 6.78 | 0.35 | * |
| Path Ellipse ( ${ }^{\circ}$ ) $\dagger$ | -1.4 | $\pm$ | 4.1 | -0.1 | $\pm$ | 2.7 | -2.7 | $\pm$ | 4.8 | 5.78 | 0.32 | * |
| Angle of Attack Actual ( ${ }^{\circ}$ ) | 1.0 | $\pm$ | 3.0 | 1.9 | $\pm$ | 2.8 | 0.1 | $\pm$ | 2.9 | 4.97 | 0.30 | * |
| Angle of Attack <br> Ellipse ( ${ }^{\circ}$ ) $\dagger$ | 1.7 | $\pm$ | 3.0 | 2.6 | $\pm$ | 2.8 | 0.8 | $\pm$ | 2.9 | 5.06 | 0.30 | * |

Table 2. Bias, root mean square error (RMSE) and normalised RMSE between the ellipse fitted impact characteristics and actual impact characteristics
$\left.\begin{array}{lrrr}\hline & \text { Bias ( }{ }^{\circ} \text { ) } & \text { Bias-Corrected } \\ \text { RMSE ( }{ }^{\circ} \text { ) }\end{array} \quad \begin{array}{r}\text { Normalised Bias- } \\ \text { Corrected RMSE (\%) }\end{array}\right]$

Figure 4. RMSE of the ellipse fitted path and angle of attack for each player


### 3.2. Group differences

Using Pillai's trace, a significant effect of skill level was found for the dependent variables $(V=1.00, F(6,45)=3.02, P<0.05)$. Univariate analysis showed that the horizontal plane angle differed significantly between skill levels $(F(1,50)=7.08, P<0.05)$, with the high skilled group angled $1.8^{\circ}$ right and the intermediate skilled group angled $2.3^{\circ}$ left (table 3). The high skilled group also had greater flattening in the fitted ellipse, with a $0.6 \%$-point difference between groups $(U=225, z=-2.06, P<0.05, r=-0.29)$.

Table 3. Group means, standard errors and statistical differences of plane and ellipse variables ( $\dagger$ denotes Mann-Whitney U test used, all others used ANOVAs, * denotes significant difference between skill levels ( $\mathrm{p}<0.05$ ))

|  | All players |  |  | High Skilled Group |  |  | Intermediate Skilled Group |  |  | Effect size of group diff |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean |  | SE | Mean |  | SE | Mean |  | SE | F | r |  |
| Horizontal Plane Angle (deg) | -0.2 | $\pm$ | 0.8 | 1.8 | $\pm$ | 0.9 | -2.3 | $\pm$ | 1.3 | 7.08 | 0.35 | * |
| Vertical Plane Angle (deg) | 50.0 | $\pm$ | 0.5 | 49.4 | $\pm$ | 0.4 | 50.7 | $\pm$ | 0.9 | 1.88 | 0.19 |  |
| Ellipse flattening (\%) $\dagger$ | 2.8 | $\pm$ | 0.2 | 3.1 | $\pm$ | 0.2 | 2.5 | $\pm$ | 0.2 | -2.06 | -0.29 | * |

## 4. Discussion

The aim of this study was to determine how well the clubhead trajectory near impact fitted to an ellipse on an inclined plane. This has been quantified and found to have minimal fitting error. It was additionally hypothesised that this ellipse would be more eccentric in high skilled golfer, and this hypothesis has been accepted.
4.1. Ellipse and plane fitting

The fitting error of 1.1 mm in the delivery plane was equivalent to previous research. The higher error of 3 mm reported by Kwon et al. (2012) compared to the present study may be due to Kwon et al. (2012) including the club-ball collision in the fitting process. Any deflection of the clubhead during this collision may have increased the fitting error. The results compare favourably to those of Morrison et al. (2014) who also only used middownswing to impact in their analysis. Both Kwon et al. (2012) and Morrison et al. (2014) suggested their values indicated a high level of planarity in their respective phases, and with even lower fitting error in the present study, planarity can be accepted as high.

The error between the fitted ellipse and the original trajectory was 1.2 mm . This is only marginally greater than the plane fitting alone. Therefore, it may be claimed that the trajectory of the clubhead in delivery follows an elliptical path on an inclined plane with some degree of accuracy. This finding confirms the work of Dunn (1934, cited in Jenkins, 2007) who originally proposed this trajectory and plane.

The intention of fitting the clubhead trajectory to an ellipse was to allow a geometric relationship to be established between the ellipse orientation and the impact characteristics. Therefore, the resultant errors in the ellipse fitted impact characteristics are of relevance. It appears in both the club path and the angle of attack, the ellipse fitted trajectory significantly overestimated the actual value at impact (tables $1 \& 2$ ). Therefore, these values could not be described as accurate. However, once corrected for bias the RMSE values for these variables were low. The normalised bias-corrected RMSE for club path and angle of attack were only $2 \%$ and $3 \%$ respectively. This suggests that the values could be precise enough to track changes in the club path and angle of attack.
4.2. Ellipse eccentricity

A significant finding is presented with respect to the shape of the ellipse. With the trajectory of the clubhead established as elliptical, a geometric relationship has been suggested between the plane orientation and the impact paths of the club. However, this relationship is dependent on the shape of the ellipse, and the fitted ellipse was found to be more eccentric in the high skilled group. The hypothesis that the ellipse would be more eccentric in the high skilled group was accepted.

In the formulation of the hypothesis it was suggested that any difference in ellipse eccentricity between skill level groups may be associated with lower variability in path and angle of attack. However, the difference between groups in real terms was very small. Assuming a short radius of the fitted ellipse of 1.15 m in both groups for illustrative purposes (slightly longer than the length of a driver), a $0.6 \%$-point difference in flattening would equate to a long radius in the intermediate group being 8 mm shorter than the high skilled group. This small difference is unlikely to have an impact on the variability in club path or angle of attack at impact.

### 4.3. Club Path, Angle of Attack and Plane Orientation

Group differences were found in all measures of club path and angle of attack. The values of club path for the two groups were very similar to those found by Betzler et al. (2012), who also found significantly higher values in the high skilled players. The values for angle of attack were also very similar to Betzler et al. (2012), although they did not find any significant differences between groups. This may have been due to the additional separation between handicap groups in the current study, where Betzler et al. (2012) used adjacent handicap groups. The high skilled group also had a horizontal plane angle further right (1.8 $\pm$ $0.9^{\circ}$ ) than the intermediate skilled group $\left(-2.3 \pm 1.3^{\circ}\right)$, and this difference was statistically significant ( $r=0.35, P<0.05$ ) (table 3 ). Plane angle has not previously been investigated with
respect to skill level, although clearly different measures to club path and angle of attack, it has been demonstrated here a relationship exists between the two variables. Betzler et al. (2012) found the path of the club pointed progressively further left in higher handicap categories, with significant differences between handicap categories 1,2 and 3. This is corroborated in the current findings in both horizontal plane angle and club path.

The club path being close to zero would indicate that the high skilled group preferred a straighter shot. While the intermediate skilled group had a club path left of the target, which would suggest a fade shot (a shot that starts left of the target and finishes on the target) was preferred. Hogan (1957) and Suttie (2005) both observed that this shape of shot was common in high handicaps, suggesting that a possible cause was the player 'casting out' their hands, wrist and arms resulting in the club being swung across the ball at impact. Whether this type of shot is associated with greater shot outcome accuracy has not been investigated to date, and is a valid line of inquiry for future research.

It is also interesting to note how the 2 groups used the orientation of the delivery plane. The high skilled group had a delivery plane that pointed right of the target; in layman's terms the direction of the swing was right of the target. However, due to these players striking the ball after the lowest point on the arc, the club path was close to zero and the angle of attack was in an upwards direction. Previously, Coleman and Anderson (2007) found that their version of swing plane was also orientated right of the target in low handicap players. They suggested that these players may have been attempting to hit a draw; however, they also suggested that the position of the ball further forward in the stance meant that the ball was contacted later in the arc. From the results presented here, it may be the case that the players were utilising the orientation of the delivery plane to hit straight shots while contacting the ball on an upward trajectory. Conversely, the intermediate skilled players struck the ball near the bottom of the arc and utilised a horizontal plane angle pointing left of the target. Making players more
aware of how these variables interact may help them to achieve more desirable impact characteristics, and the information gained here can assist coaches in doing so.

The vertical plane angle did not appear to differ significantly between groups. Another suggestion of Dunn was that the vertical incline of this swing plane was determined by the player's height (Jenkins, 2007). As in this study the height of the two groups were not significantly different, it follows that the vertical plane angles would also not differ. These values were also comparable to Kwon et al. (2012), who found that this vertical plane angle increased with shorter clubs. In the current study the vertical plane angle ranged from 43 and 60 degrees, and the following section will demonstrate how these extremes can have an influence on the impact characteristics. Further research regarding anthropometrics and vertical swing plane should be carried out to ascertain if any relationship exists, or if it is a changeable element of technique.

### 4.4. Coaching implications

The current findings have implications for golf coaches in their analysis of the golf swing. As an alterable aspect of technique, it is important for coaches to understand how alterations in the swing plane will affect the result of the shot. The impact characteristics represent the last changeable factor in the golf swing and have a direct effect on the shot outcome (Betzler et al., 2014). The current results allow for a relationship to be defined between the swing plane orientation and the club path and angle of attack, two impact characteristics that have a direct effect on the shot outcome. For a given angle of attack (AofA), vertical plane angle ( $V P A$ ) and horizontal plane angle (HPA), club path (Path) would be calculated as follows:

$$
\begin{equation*}
\text { Path }=\tan ^{-1}\left(\frac{\tan (-A o f A)}{\tan ^{2}(V P A)}\right)+H P A \tag{3}
\end{equation*}
$$

However, coaches are unlikely to use this complex equation in their practice. A 'rule of thumb' may be more useful for practical application. Taking into account the likely range of values for club path, angle of attack and vertical swing plane, the relationship becomes almost linear (figure 5).

Table 4. 'Rule of thumb' figures for the relationship between club path, angle of attack, and vertical and horizontal plane angle

| Vertical plane angle <br> (degrees) | Horizontal plane <br> angle (degrees) | Club path (degrees) | Angle of attack <br> (degrees) |
| :---: | :---: | :---: | :---: |
| 45 | 0 | 1.0 | -1.0 |
| 55 | 0 | 1.0 | -2.0 |
| 65 | 0 | 1.0 | -4.5 |

Figure 5. 'Rule of thumb' plots of club path vs angle of attack for a range of vertical plane angles and horizontal plane angle of zero (VPA = vertical plane angle)


For example, in a hypothetical swing with a horizontal plane angle of zero and a vertical plane angle of 45 degrees, a ball struck early or later on the circular arc would have equal effects on the angle of attack and club path, i.e. a club path pointing 1 degree further right of the target would be accompanied by an angle of attack 1 degree more downward (table 4). However, if the player's vertical plane angle were 55 degrees, a club path pointing 1 degree further right would be accompanied by an angle of attack approximately 2 degrees more downward (table 4). This information can help coaches in their decision making when attempting to change the club path or angle of attack of a player. For instance, Jenkins (2007) suggested that the height of a player might affect the vertical angle of the plane. A coach working with a taller player should be aware that changes in impact location relative to the low point of the swing arc may have different effects on club path and angle of attack than if working with a shorter player.

While it is necessary in biomechanics to seek accuracy in the measurements and calculations that are made, the immediacy required in a practical coaching setting may mean simpler calculations are merited. Using these 'rule of thumb' values may be more applicable to coaches.

## 5. Conclusion

The trajectory of the clubhead leading up to ball impact was analysed and the results indicated that the clubhead trajectory fitted with minimal error to an ellipse on an inclined plane. The hypothesis that the fitted ellipse would be more eccentric in the high skill level golfers was accepted. With the ellipses only displaying slight eccentricity, coaches may be able to assume a circular trajectory when explaining the relationship between the orientation of the delivery plane and the club path and angle of attack at impact. The relevance of the
delivery plane in the golf swing has been shown, which provides a novel method for further research into the relationship between technique and shot outcome.

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