

Grading entropy coordinates and criteria for evaluation of liquefaction potential

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ABSTRACT: Several methods are commonly used for the evaluation of liquefaction potential in soils. These include criteria based on SPT and CPT tests, shear wave velocity measurements and gradation limits, amongst others. This study applies the concept of grading entropy coordinates that enables plotting any particle size distribution (PSD) as a single coordinate pair in a Cartesian plane, whilst also considering the entire range of the grading curve. This is in contrast to descriptors such as the mean particle diameter (d_{50}) and the coefficients of uniformity and curvature, c_u and c_c , respectively, which ignore the fines content as well as the presence of coarser particles within the PSD. Considering 62 gradings identified as liquefiable soils in existing studies it was demonstrated that the grading entropy coordinates are a potentially effective tool to assess liquefaction potential. Furthermore, it was postulated that existing internal stability criteria in terms concept of grading entropy coordinates may also be used as a framework for the evaluation of liquefaction potential.

1 INTRODUCTION

The evaluation of liquefaction potential of soils is a key component of any site investigation for construction projects in earthquake prone areas, as well as those involving foundation vibration due to machinery and equipment. Common methods to perform liquefaction potential assessments vary and include those based on:

- (i) measured blow counts from the Standard Penetration Test -SPT- (e.g. Seed & Idriss, 1971; Youd *et al*, 1997);
- (ii) Cone Penetration Test – CPT- results (e.g. Robertson and Campanella, 1985; Olsen, 1997; Robertson & Wride, 1998),
- (iii) in-situ and/or laboratory measurements of small strains shear wave velocity (e.g. Tokimatsu & Uchida, 1990; Andrus & Stokoe, 2000)
- (iv) particle gradations (e.g. Tsuchida & Hayashi, 1971; Numata & Mori, 2004).

Each of these methods has its own merits and shortcomings. It may be argued that SPT and CPT based methods are somehow related to the shear strength of soils and shear-wave velocity methods, which rely on the shear stiffness of soils. Both shearing resistance and shear stiffness are often linked to and/or are dependent on the characteristics of particle size distributions (e.g. Wichtmann and Triantafyllidis, 2009; Casini *et al*, 2011), hence evaluation liquefaction potential based on gradation characteristics may incorporate all other recognized effects and dependencies on initial void ratio, stress level, stress history, etc. However, as in the case of the gradation curves limits proposed by Tsushida and Hayashi (1971) the criteria for identification of soil that are potentially liquefiable or most likely to be liquefied are not always fulfilled and gradations outside their limits have been shown to liquefy (e.g. Ishihara, 1997; Ward *et al* 2001, Numata & Mori, 2004).

Existing systems in terms of gradings (for liquefaction potential) are based on traditionally used descriptors of particle size distributions (PSDs) such as the mean size diameter (d_{50}) and

the coefficient of uniformity (c_u), amongst others. In fact the proposal by Tsushida & Hayashi (1971) proposed different gradation limits for poorly graded soils ($c_u < 3.5$) and well-graded soils ($c_u > 3.5$) based on a database of 2051 soils which were known to have developed liquefaction in past earthquakes. Despite the abundance of empirical data, such approach may be problematic because it does not recognize the effect of the fines content (FC) on the occurrence of liquefaction. The effect of FC on soil liquefaction is however well recognized (e.g. Thevanayagan, 1998; Wei & Yang, 2014; Paydar & Ahmadi, 2016; Zhou *et al*, 2018).

This study aims to present an innovative method for the quantitative classification of gradation curves based on the whole range of sizes represented by it. The method proposed by Lőrincz (1982) is derived from the definition of the statistical entropy of a discrete distribution and enables to represent any PSD by a single point on a Cartesian (x-y) plane. Such representation of grading curves has enabled to further understand the evolution of PSD due to particle breakage (Lőrincz *et al*, 2005), and mineral dissolution (McDougall *et al*, 2013), amongst other geotechnical processes. More relevant to this study, the concept of grading entropy and the results of suffusion tests on soils with diverse gradings have been linked to establish an internal stability criterion (Lőrincz *et al*, 2015). It is postulated here that for liquefaction the same criterion can be used because it is based on the inherent stability of soil grains, which is in turn affected by the particle size distribution.

2 STATISTICAL ENTROPY, GRADING ENTROPY AND RELATED INTERNAL STABILITY CRITERION.

2.1 Statistical entropy

Entropy has been classically defined within the context of a number of specific applications. For example the classical entropy of thermodynamics (S_c), the statistical formulation of the classical entropy of thermodynamics (S_t), the entropy of information theory (S_i), and the statistical entropy (S_s). For the context of this study the definition used is that of the statistical entropy of a discrete distribution function (S_s). This definition has an implicit link with the principles of thermodynamics, however it is more easily understood if it is firstly presented as a purely mathematical concept. Considering a discrete statistical distribution with M elements in m equal cells/bins, where M_i is the number of the elements in the i^{th} cell/bin, the statistical entropy S_s is given by:

$$S_c = Ms \quad (1)$$

where s is the specific entropy, or the entropy of an element given by:

$$s = - \sum_{i=1}^m \alpha_i \log_b \alpha_i \quad (2)$$

where b is the base of the logarithm and α_i is the frequency of the i^{th} cell/bin given by:

$$\alpha_i = \frac{M_i}{M} \quad (3)$$

2.2 Grading entropy

Referring to Equation 2, Lőrincz (1986) has chosen the base of the logarithm (b) in such way that for the case of two statistical cells (i.e. only two sieves sizes used during the experimental measurement of the particle size distribution) the maximum entropy is 1 and for this case the relative frequencies are also equal to $\alpha_1 = \alpha_2 = 0.5$. In other words, a PSD with mass retained in only two of the sieves used with percentage of mass retained equal to 50% for both sieves produces a maximum value of entropy equal to 1. Hence, when $b = 2$ in Equation 2, it becomes:

$$s = - \frac{1}{\ln 2} \sum_{i=1}^m \alpha_i \ln \alpha_i \quad (4)$$

The grading curve is measured generally by dry and/or wet sieving and a variety of sieve sizes may be used. For the calculation of grading entropy coordinates, the sieve mesh diameters usually increase by a multiple of 2 (e.g. 0.25, 0.5, 1, 2, 4, 8 mm). The diameter range for fraction j ($j = 1, 2, \dots, j$, see Table 1) is:

$$2^j d_0 \geq d > 2^{j-1} d_0 \quad (5)$$

where d_0 is the smallest diameter which is equal to a theoretical minimum grain size: the height of the SiO_4 tetrahedron (Imre, 1995). These statistical cells do not restrict the use of grading entropy coordinates according to international testing standards. It means that for the calculation of the statistical entropy, the values of the mass percentages retained in the sieves with sizes of 0.25, 0.5 mm, etc. are used. In practical terms and considering that PSDs in soils may vary across several orders of magnitude in terms of the particle diameter (d), the increments in d (by a multiple of 2) are separated in the standard granulometric curve by an equal distance as illustrated by the discontinuous lines in Fig. 1a. Hence, the PSD graph of any soil can be quantified with values evenly distributed across the whole range of particle diameters represented.

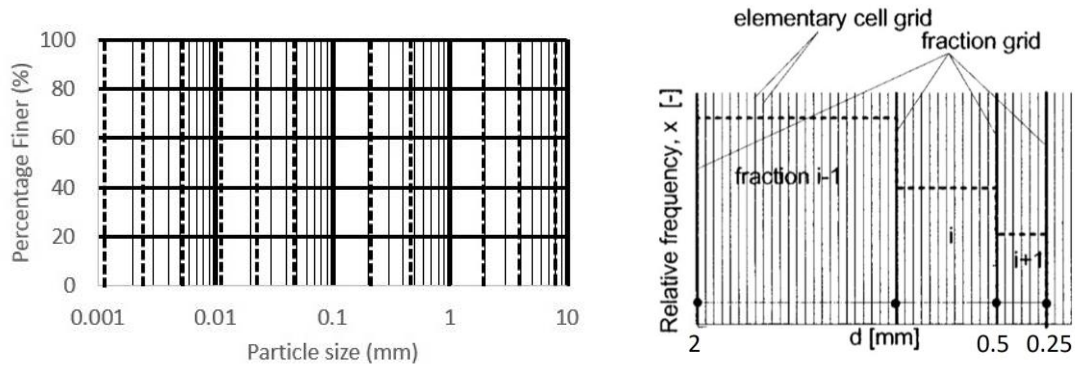


Figure 1. (a) Particle size distribution axes showing particle sizes divided in increments of 2. (b) the fraction and elementary cell systems (modified after Lőrincz et al, 2005)

Table 1. Fractions, their numbering and equivalent eigenentropies (S_{0j})

j	1	23	24
Limits	d_0 to $2 d_0$	$2^{22} d_0$ to $2^{23} d_0$	$2^{23} d_0$ to $2^{24} d_0$
S_{0j} [-]	1	23	24

It is convenient to consider that the cells/bins in Equations 2 and 3 have uniform size in the statistical entropy concept. Since the fraction diameter limits are doubled, this primary cell system is refined into a uniform cell system for the derivation of the grading entropy. In other words, in the grading entropy concept a double statistical cell system is used (Fig. 1b). The primary cell system, referred to as the fraction cell system corresponds to specific sieves (with size increments with multiples of 2, Table 1) used for the standard measurement and calculation of particle size distribution curves in soils. The (second) elementary cell system comprises cells are with uniform size equal to the height of the SiO_4 tetrahedron (i.e. the theoretical minimum grain size). A full derivation of the grading entropy coordinates has been published elsewhere, taking into account the double statistical cell system (e.g. Lőrincz *et al*, 2005). It is however important to highlight that Equation 4 can be presented as a sum of two terms as follows:

$$S = \Delta S + S_0 \quad (7)$$

where S is the grading entropy, ΔS is the entropy increment and S_0 is the base entropy given by:

$$\Delta S = -\frac{1}{\ln 2} \sum_{i=1}^N x_i \ln x_i \quad (8)$$

$$S_0 = \sum_{i=1}^N x_i S_{0i} \quad (9)$$

where x_i is the relative frequency of the fraction i , ($i=1..N$), S_{0i} is the eigenentropy of the fraction i , ($i=1..N$). Referring to Table 1, for the special case of a PSD consisting of a single fraction (i.e. soil mass only retained in one sieve, hence $N=1$) it follows from Equations 7 to 9 that the grading entropy of the soil (S) is a natural number, referred to as the “eigenentropy” of the i^{th} fraction (S_{0i}). Both ΔS and S_0 can be normalized to obtain the normalized entropy increment (B) and the relative base entropy (A), respectively:

$$B = \frac{\Delta S}{\ln N} \quad (10)$$

$$A = \frac{S_0 - S_{0\min}}{S_{0\max} - S_{0\min}} \quad (11)$$

where $S_{0\min}$ and $S_{0\max}$ are eigenentropies of the smallest and largest fractions in the mixture, respectively. The coordinate pair (A , B) can then be used to represent any particle size distribution as a single point. The relative base entropy (A) is a logarithmic mean of the particle size and is somehow related to the mean particle diameter (d_{50}). On the other hand, the normalized entropy increment (B) relates to the fractions that are represented in the PSD. Both coordinates (A and B) also implicitly relate to the coefficients of uniformity (c_u) and curvature in terms of the kurtosis and skewness of the PSD as illustrated in Figure 2. Note that the entire particle size distribution is considered in the definition of the entropy coordinates (A , B) unlike the case of common descriptors such as d_{50} , c_u and c_c .

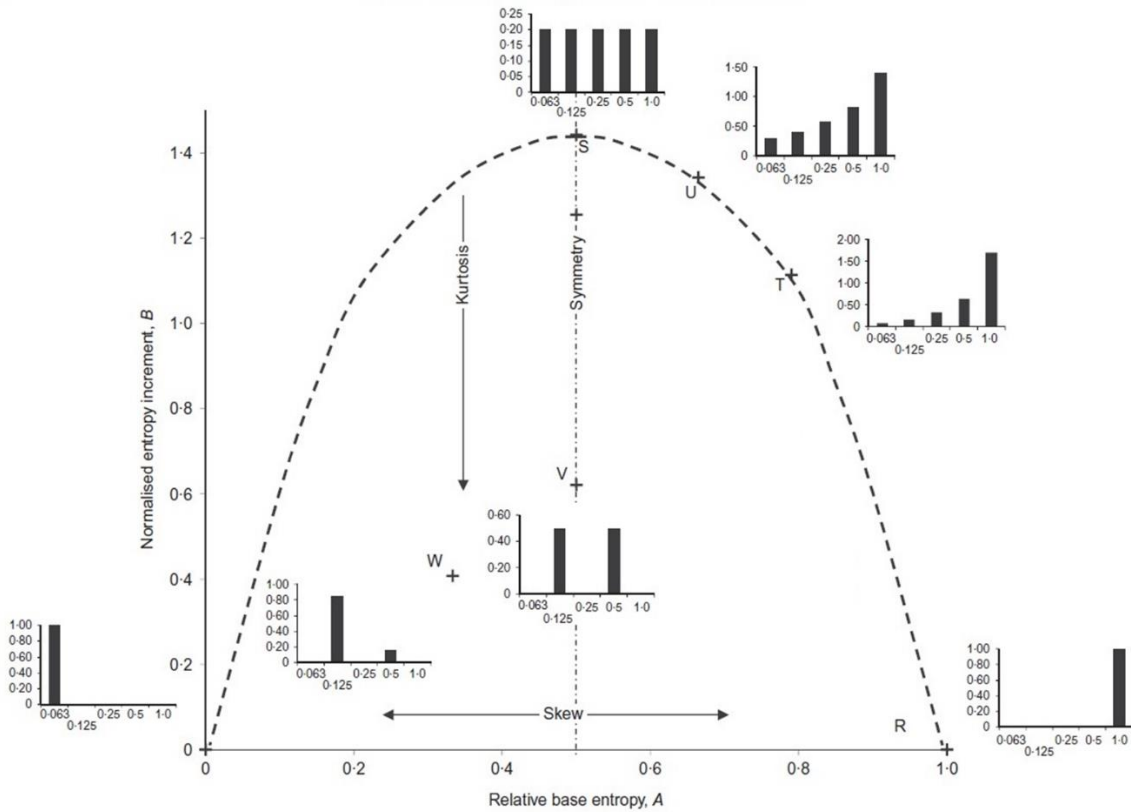


Figure 2. The normalized entropy diagram showing typical particle size distributions (modified after McDougall *et al*, 2013)

A few important remarks can be made regarding the grading entropy coordinates with reference to the normalized entropy diagram illustrated in Fig. 2. The horizontal axis represents the values of relative base entropy (A) and varies between 0 and 1. A PSD with a single fraction of very small size will be represented at the lower left corner of the diagram ($A=0$, $B=0$). Similar-

ly, a single fraction PSD with large mean particle diameter is represented by the point (1,0) at the lower right side of the diagram. The relative base entropy is related to the mean particle size (d_{50}). The normalized entropy increment (B) has a maximum value of approximately 1.442 when $A = 0.5$ and this value of A represents all the uniformly distributed particle size distributions (with values of relative fractions equal for all sieve sizes) and the value of B increases as a function of the number of fractions/sieves where some material has been retained. Note that PSDs on the right side of the diagram (such as those represented by points R, T and U) have a “dominant” fraction towards the largest fraction and the opposite happens at the left side of the diagram where the largest “dominant” relative frequency occurs for a smaller sieve.

2.3 Internal stability criterion in terms of grading entropy coordinates

The entropy parameters were used when the internal stability rule was elaborated on the basis of suffusion test data measured for well-designed sand mixtures performed by Lőrincz (1986). On the basis of these test results, three basic types of soil structures were related to three domains of the normalized entropy diagram (see Fig. 3). Of particular interest is the division between unstable and (transitionally) stable zones in terms of the particle size distribution (when $A = 2/3$). It is also important to highlight that this criterion is compatible with the framework proposed by Thevanayagam (1998). In Zone 1 (unstable) with $A < 2/3$ the coarse particles “float” in a matrix of the fines and become destabilized when the fines are removed by erosion/piping. In the (transitionally) stable Zone 2 (and 3), a stable structure is present.

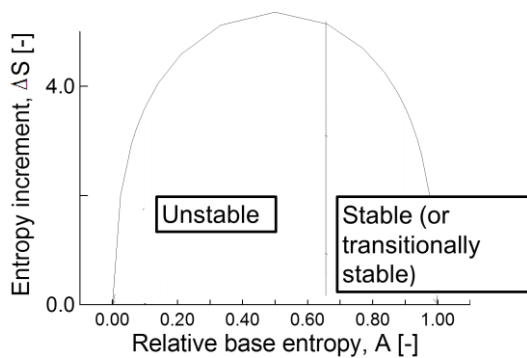


Figure 3. Internal stability criterion in the (partly) normalized entropy diagram

The physical meaning of the stability criterion is that the relative base entropy (A) relates the proportion of large grains, which comprise a stable fabric if they are present in a large enough quantity. With reference to observations from DEM simulations, A relates to the number of “rattlers” (i.e. particles not carrying a significant proportion of stress). As A decreases, the number of rattlers increases. In other words, as A increases the number of particles carrying a large proportion of the stress increases, generating an inherently more stable structure (Imre et al, 2018).

3 GRADING ENTROPY COORDINATES AS AN ADDITIONAL TOOL FOR THE EVALUATION OF LIQUEFACTION POTENTIAL

For preliminary purposes a database consisting of 62 different gradings that have been reported in the literature as liquefied have been considered here. The database comprises both laboratory tests and in-situ measurements after earthquakes, as well as a variety of sands with a wide range of mean particle diameters and coefficients of uniformity. Figure 4 shows each of the particle sizes distributions represented as a single point by means of the grading entropy coordinates. For comparison purposes the gradation limits proposed by Tsushida and Hayashi (1971) have been plotted as circles. Interestingly for both poorly and well-graded soils most of these limit gradations plot on area of the entropy diagram where the entropy value is maximum (i.e. $A \cong$

0.5 and $B \rightarrow 1.442$). Notably, the cyclic triaxial experiments by Chang & Ko (1982) include some gradations that would be classified by the criteria of Tsushida and Hayashi (1971) as non-liquefiable soils. A similar comment can be made in relation to the experimental data by Zhou et al (2010) obtained from centrifuge tests. Notably, one of the gradations of (liquefied) Earls Creek sand tested by Vaid et al (1990) plots well outside the range of the other data due to its significant content of fines. Numerical (DEM) data on sphere specimens by Zhou et al (2017) that were reported as liquefiable also plot close to the maximum entropy area and within the limits established by Tsushida and Hayashi (1971).

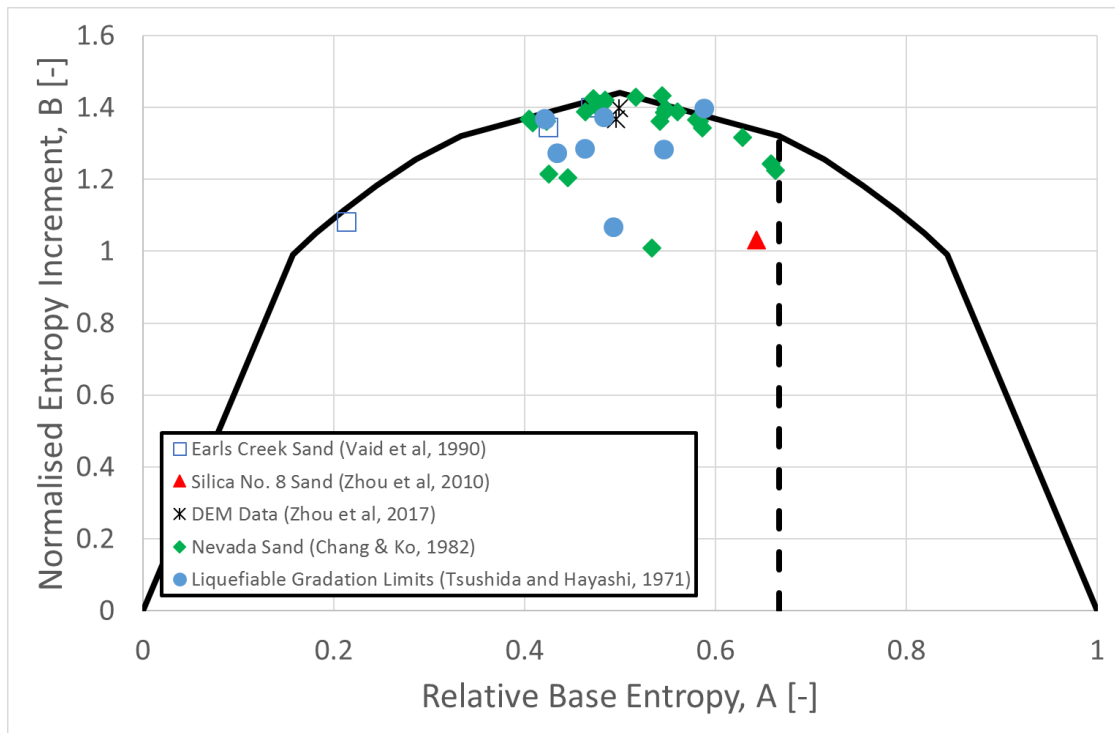


Figure 4. A variety of liquefied soils and their corresponding particle size distributions represented by a single point by means of the grading entropy coordinates.

Perhaps of the outmost interest is that all data points plot to the left of the $A = 2/3$ line related to the internal stability criterion described on the previous section. Hence, it may be concluded that the internal stability criterion also relates to the susceptibility to liquefaction in as much as the shear strength of soils in both suffusion, piping and erosion processes, as in liquefaction, relates to the inherent stability of soils grains and their ability to form strong force chains that are able to support a larger proportion of the major principal stress.

4 DISCUSSION AND CONCLUSIONS

This study has considered available data from undrained cyclic triaxial experiments, centrifuge tests, as well as field data from post-earthquake investigations and numerical data from DEM simulations. Using the principles of statistical entropy, and in particular the definition of grading entropy coordinates we have represented a total of 62 gradations on the normalized entropy diagram. Perhaps unsurprisingly but interestingly all data locates within the “unstable” zone of the internal stability criterion defined by Lőrincz (1986) on the basis of suffusion tests (i.e. where $A < 2/3$). Hence, it has been postulated that liquefaction potential obeys the same physical laws that govern suffusion, piping and internal erosion phenomena. Furthermore, we have highlighted that grading entropy coordinates are more effective on identifying potentially liquefiable

soils than other existing criteria based on gradation limits such as that proposed by Tsushida and Hayashi (1971).

Clearly, additional research is warranted to further understand the effects of the PSD on other properties, such as initial density, number of cycles required for liquefaction, small strain shear stiffness, etc. In particular from the perspective of the grading entropy coordinates. Some preliminary research has been performed to link grading entropy coordinates and the value of the small-strain shear stiffness (Barreto et al, 2018) and further work to gain further understanding, and establish stronger links between grading entropy and existing criteria for the evaluation of liquefaction potential should be made. This is however out of scope for this study.

REFERENCES

- Andrus, R.D. & Stokoe II, K.H. 2000. Liquefaction resistance of soils from shear-wave velocity. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 126(11): 1015-1025
- Barreto, D and Imre, E. 2018. Grading entropy coordinates and their relationship to shear stiffness in granular materials. *Proceedings of IS Atlanta 2018 Symposium on Geomechanics from Micro to Macro in Research and Practice*, Atlanta, in press.
- Casini, F., Brauchli, S., Herzog, R., & Springman, S. 2011. Grain size distribution and particle shape effects on shear strength of sand – gravel mixtures. *In Proceedings of the 15th European Conference on Soil Mechanics and Geotechnical Engineering*, A. Anagnostopoulos et al. (Eds). 149-154
- Chang, N.Y. & Ko, H.Y. 1982. Effects of gran size distribution on dynamic properties and liquefaction potential of granular soils. Research Report R82-103, Geotechnical Publication 111 under Grant No. PFR 78-23094 of the National Science Foundation.
- Imre, E. 1995. Characterization of dispersive and piping soils. *Proc.11th ECSMFE*, Vol. 2, Copenhagen, Denmark, 49–55.
- Imre, E., Barreto, D., Talata, I., Goudarzy, M., Rahemi, N., Baille, W. & Singh, V.P. 2018. Fractal and optimal gradings and their relationship to internal stability. *Proceedings of IS Atlanta 2018 Symposium on Geomechanics from Micro to Macro in Research and Practice*, Atlanta, in press.
- Ishihara, K. 1997. Terzaghi oration: Geotechnical aspects of the 1995 Kobe earthquake. *Proceedings of 14th Int. Conf. Soil Mech. & Found. Eng.* 2047-2073
- Lőrincz, J. 1986. *Grading entropy of soils*. PhD Thesis, Tech Sc, Technical University of Budapest (in Hung).
- Lőrincz, J., Imre, E., Gálos, M.,Trang, Q.P., Rajkai, K., Fityus, S. & Telekes, G. 2005. Grading Entropy Variation Due to Soil Crushing. *International Journal of Geomechanics*, ASCE, 5(4): 311-319
- Lőrincz, J.; Imre, E.; Fityus, S.; Trang, P.Q.; Tarnai, T.; Talata, I. & Singh, V.P. The Grading Entropy-Criteria for Stability of Granular Materials and Filters. *Entropy*, 17, 2781-2811.
- McDougall, J.R., Imre, E., Barreto, D. & Kelly, D. 2013. Volumetric consequences of particle loss by grading entropy. *Géotechnique*, 63(3): 262-266
- Numata, A. & Mori, S. 2004. Limits in the gradation curves of liquefiable curves. *In 13th World Conference on Earthquake Engineering*, Vancouver, B.C., Paper No. 1190
- Olsen, R.S. 1997. Cyclic liquefaction based on the cone penetrometer test. *Proc. NCEER Workshop on Evaluation of Liquefaction Resistance of Soil. Tech. Rep. NCEER-97-0022*.
- Paydar, N.A. & Ahmadi, M.M. 2016. Effect of Fines Type and Content of Sand on Correlation Between Shear Wave Velocity and Liquefaction Resistance. *Geotech Geol Eng* 34:1857–1876
- Robertson, P.K. & Campanella, R.G. 1985. Liquefaction potential of sands using the CPT. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 11(3): 384-403
- Robertson, P.K. & Wride, C.E. 1998. Evaluating cyclic liquefaction potential using the cone penetration test. *Canadian Geotechnical Journal*, 35(3): 442-459
- Seed, H.B. & Idriss, I.M. 1971. Simplified procedure for evaluating soil liquefaction potential. *J. Soil Mech and Found. Div.*, ASCE, 97(9): 1249-1273
- Thevanayagam, S. 1998. Effects of fines and confining stress on undrained shear strength of silty sands. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 124(6): 479-491
- Tokimats, K. & Uchida, A. 1990. Correlation between liquefaction resistance and shear wave velocity. *Soils and Foundations*, 30(2): 33-42
- Tsuchida, H. & Hayasi, S. 1971. Estimation of liquefaction potential of sandy soils. Estimation of liquefaction. *In Proceedings of the Third Joint Meeting, US-Japan Panel on Wind and Seismic Effects*, UJNR, Tokyo, May 1971, pp. 91-109.
- Vaid, Y.P., Fisher, J.M., Kuerbis, R.H. & Negussey, D. 1990. Particle gradation and liquefaction. *Journal of Geotechnical Engineering*, 116(4): 698-703

- Ward, S.D., Brown, M.K.H. & Brown, I.R. 2001. Geological engineering study of liquefaction after the 2010 Darfield earthquake in an area of complex fluvial geology. In Proceedings of the Ninth Pacific Conference on Earthquake Engineering Building an Earthquake-Resilient Society, Auckland, New Zealand. Paper No. 053.
- Wei, L. M. & Yang, J. 2014. On the role of grain shape in static liquefaction of sand–fines mixtures. *Geotechnique*, 64(9): 740–745
- Wichtmann, T. & Triantafyllidis, T. 2009. Influence of the Grain-Size Distribution Curve of Quartz Sand on the Small Strain Shear Modulus G_{max} . *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 135(10): 1404-1418.
- Youd, T.L., Idriss, I.M., Andrus, R.D., Arango, I., Castro, G., Christian, J.T., Dobry, R., Liam Finn, W.D., Harder Jr, L.F., Hynes, M.E., Ishihara, K., Koester, J.P., Liao, S.S.C, Marcuson III, W.F., Martin, G.R., Mitchell, J.K., Moriwaki, Y., Power, M.S., Robertson, P.K., Seed, R.B. & Stokoe II, K.H. 2001. Liquefaction resistance of soils: Summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 127(10): 817-833
- Zhou, C., Xu, X.M. & Cheng, Y.P. 2017. Effect of particle size distribution on the correlation between liquefaction resistance and shear wave velocity of granular soils. In *Powders and Grains*, EPJ Web of Conferences 140
- Zhou, W. Wu, W., Ma, G., Ng, T.T. & Chang, X. 2018. Undrained behavior of binary granular mixtures with different fines contents. *Powder Technology*, 340: 139-153
- Zhou, Y.G., Chen, Y.M. & Shamoto, Y. 2010. Verification of the Soil-Type Specific Correlation between Liquefaction Resistance and Shear-Wave Velocity of Sand by Dynamic Centrifuge Test. *Journal of Geotechnical and Geoenvironmental Engineering*, 136(1): 165-177